Universidade Federal do Rio de Janeiro
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Spatial dependence of the distribution of current in nanotubes and nanowires

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Collaboration

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- **Jorge A. Otálora (UTFSM)**
- **Patricio Vargas (UTFSM)**

Journal of Applied Physics (2011)

Nanotubes and nanowires

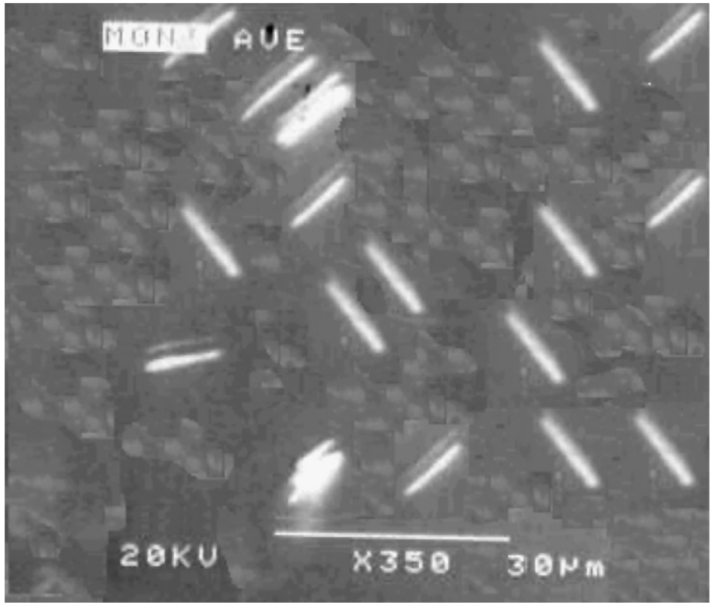


Fig. 2. SEM image of Ni nanowires.

Muosa M.A. Imran
Journal of Alloys and Compounds 455 (2008) 17–20

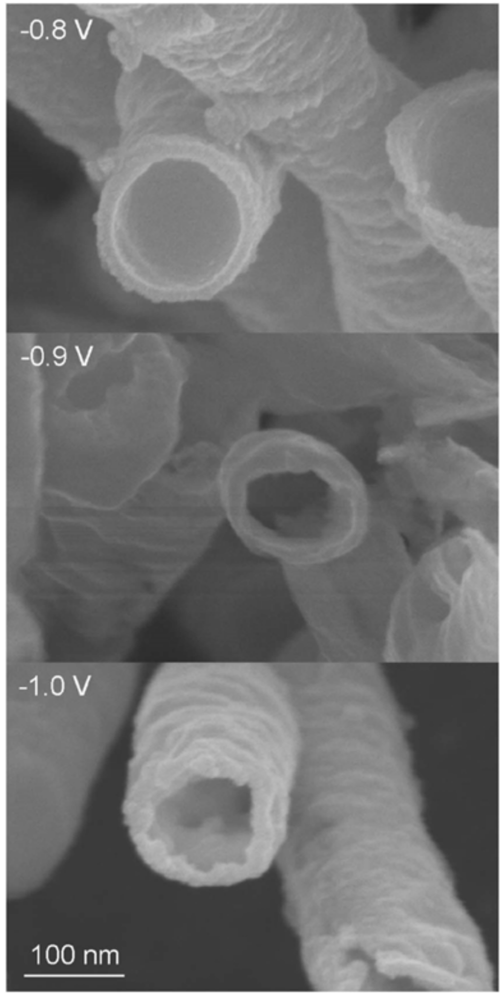


FIG. 8. SEM images of the Ni nanotubes formed by etching Ni–Cu alloy nanowires deposited at (a) −0.8 V, (b) −0.9 V, and (c) −1.0 V from solution containing 50 mM Cu(II) and 400 mM Ni(II).

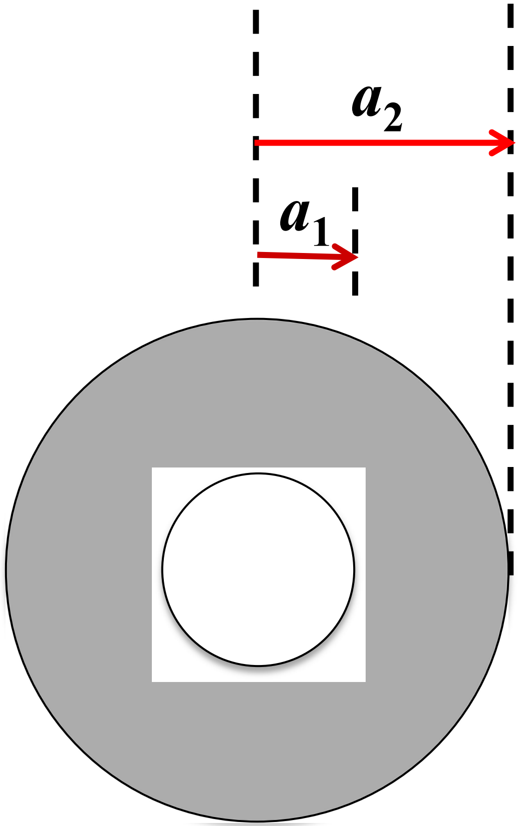
Zhu Liu et al. J. APPL. PHYS. 103, 064313 (2008)

Nanotubes and nanowires

Interesting physical properties:

- strong shape anisotropy ➔ bits of memory
- significant GMR effect
  14% in Co/Cu/Co multilayered nanowires
  Blondel *et al.* Appl. Phys. Lett. 65, 3019 (1994)
- large surface area to volume ratio ➔ biological and chemical sensors
- variety of materials ➔ metallic, semiconductor, insulator

Electronic transport in nanotubes



Electric and magnetic fields parallel to the tube axis

➔ **spatial distribution of the current within the tube/wire**

Degree of precision in the fabrication processes: dispersion in the wall thickness ~ 3 nm (iron oxide nanotubes)

J. Escrig et al., Phys. Rev. B. **77**, 214421 (2008)
J. Escrig et al., Nanotechnology **18**, 445706 (2007).

Electronic transport in nanotubes

Hamiltonian:

$$H = \frac{1}{2m^*}\left[p_a^2 + \frac{1}{a^2}\left(p_\phi - \frac{qB}{2c}a^2\right)^2 + p_z^2\right] + V,$$

where

$$V = \begin{cases} -qE_0 z, & a_1 < a < a_2 \\ \infty, & \text{otherwise} \end{cases}$$

cylindrical coordinates: $(a, \phi, z, p_a, p_\phi, p_z)$

$p_\phi =$ ***constant of motion***

Electronic transport in nanotubes

Boltzmann equation (relaxation time approximation):

$$\{H, f\} = -\frac{f - f^0}{\tau}$$

$f = distribution\ fucntion$

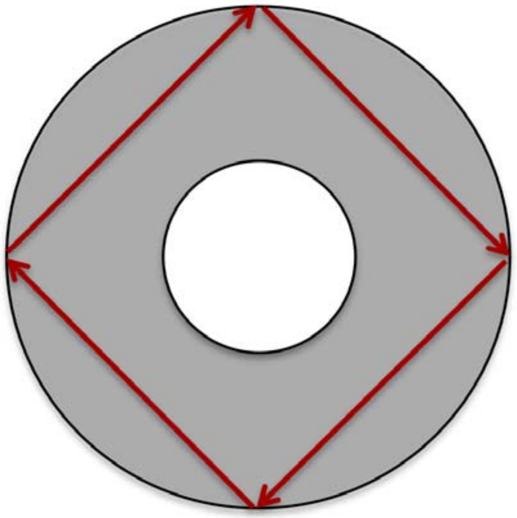
$\tau = relaxation\ tome$

$f^0 = equilibrium\ distribution\ function\ (for\ E_0 = 0)$

$= f_{FD}(\varepsilon)$

$$\varepsilon = \frac{1}{2m^*}\left[p_a^2 + \frac{1}{a^2}\left(p_\phi - \frac{qB}{2c}a^2\right)^2 + p_z^2\right]$$

Closed projections of the orbits



- Closed projections even for B = 0
- de Haas-van Alphen and the Shubnikov-de Haas effects

$$Sommerfeld-Wilson \Rightarrow p_{\phi} = \nu\hbar$$

Solution to Boltzmann equation

- **We write:** $f = f^0 + g$

- **Equation for *g***

$$\left\{ \frac{p_a}{m^*}\frac{\partial}{\partial a} + \left[\frac{\nu^2 h^2}{m^* a^3} - \frac{a}{m^*}\left(\frac{qB}{2c}\right)^2 \right]\frac{\partial}{\partial p_a} + \frac{1}{\tau} \right\} g = (qE_0)\frac{\partial f^0}{\partial p_z}$$

- **Solution (for sufficiently low temperatures)**

$$g_\nu(a, p_a, p_z) = \frac{qE_0 \tau p_z}{m^*}\delta(\varepsilon - \varepsilon_F)\left[1 + F\exp\left(\zeta_\nu(a, p_a)\right)\right]$$

Solution to Boltzmann equation

- **Boundary condition (charge conservation):**

$$g_\nu(a, p_a < 0, p_z) = g_\nu(a, p_a > 0, p_z) \quad \Rightarrow \quad F = 0$$

$\Rightarrow$

$$g_\nu(a, p_a, p_z) = \frac{qE_0 \tau p_z}{m^*} \delta(\varepsilon - \varepsilon_F)$$

$\Rightarrow$

$$j_z(a) = \frac{q}{8\pi^3 h^2} \frac{1}{a} \iint dp_a \, dp_z \sum_\nu \left(\frac{p_z}{m^*} \right) g_\nu(a, p_a, p_z)$$

Density of current

$$j_z(a, B) = \frac{q^2 E_0 \tau}{8\pi^2 h^2 m^* a} \sum_{\nu} P_{\nu}^2.$$

$$P_{\nu}^2 = 2m^* \varepsilon_F - \left(\frac{\nu\hbar}{a} - \frac{qBa}{2c} \right)^2$$

$$-\left(k_F a - \frac{qBa^2}{2\hbar c} \right) \leq \nu \leq \left(k_F a + \frac{qBa^2}{2\hbar c} \right)$$

$$k_F = \sqrt{2 m^* \varepsilon_F} / \hbar$$

Results

- **We rewrite j_z as:**

$$j_z(a,B) = \frac{j_F}{k_F a} \sum_{\nu_{\min}}^{\nu_{\max}} \left(1 - \left(\frac{\nu}{k_F a} - \frac{B}{B_F} k_F a \right)^2 \right)^2$$

$$j_F = q^2 E_0 \tau k_F^3 / 8\pi^2 \hbar^2 m^*$$

$$B_F = 2c\hbar k_F^2 / q$$

- **Numerical estimate: $n \approx 10^{16}$ cm^{-3}**

$1/k_F \approx 1.7 \times 10^2$ nm **$B_F \approx 4.7$ T**

Results

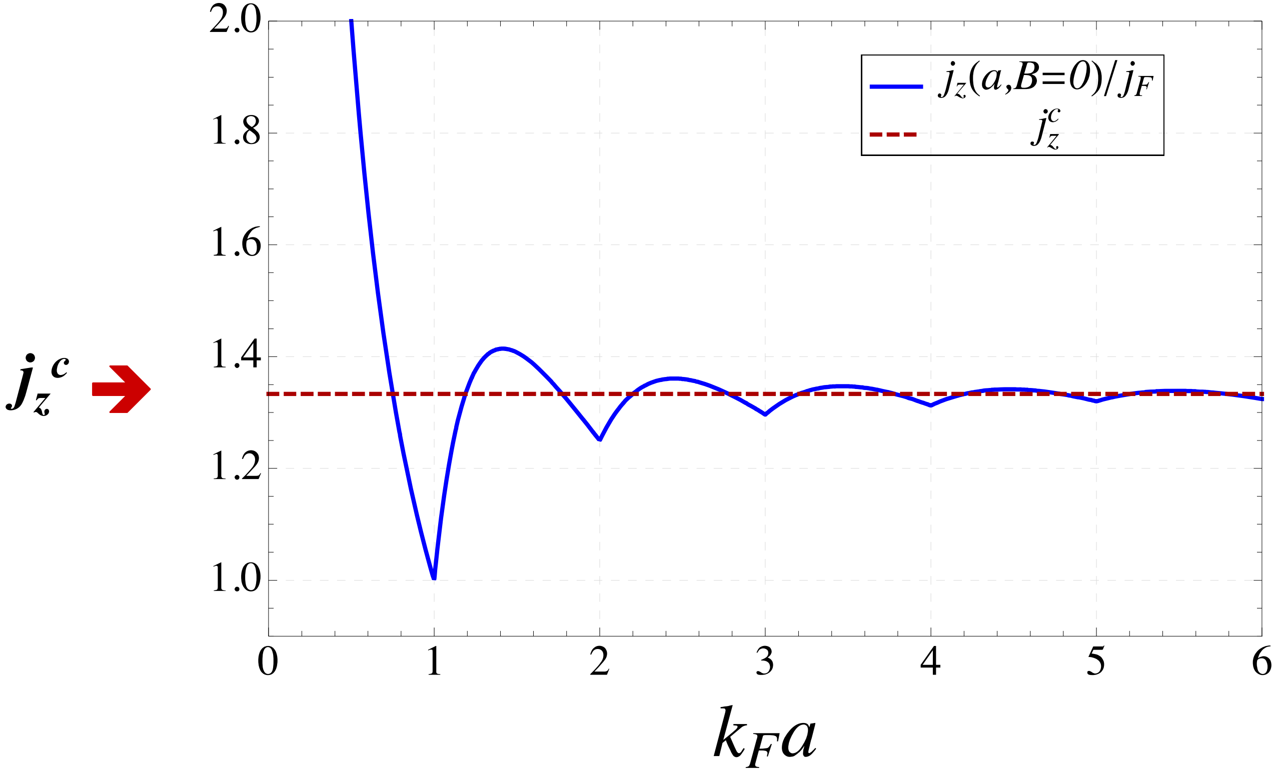
Classical result

$$j_z^c(a,B) = \frac{j_F}{\hbar k_F a} \int_{p_1}^{p_2} dp_\phi \left(1 - \left(\frac{p_\phi}{k_F a} - \frac{B}{B_F} k_F a \right)^2 \right)^2$$

$$p_1 = \hbar \left(1 - (B/B_F) k_F^2 a^2 \right) \qquad p_2 = \hbar \left(1 + (B/B_F) k_F^2 a^2 \right)$$

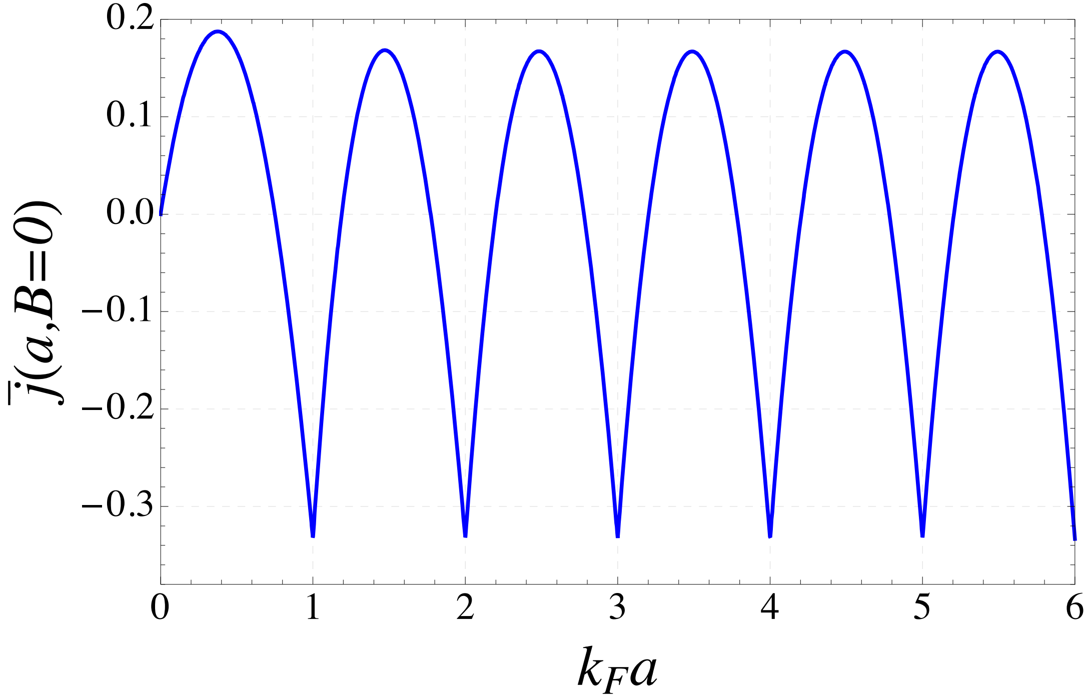
$$\Rightarrow \quad j_z^c(a,B) = (4/3)\, j_F$$

Results



B = 0

Results



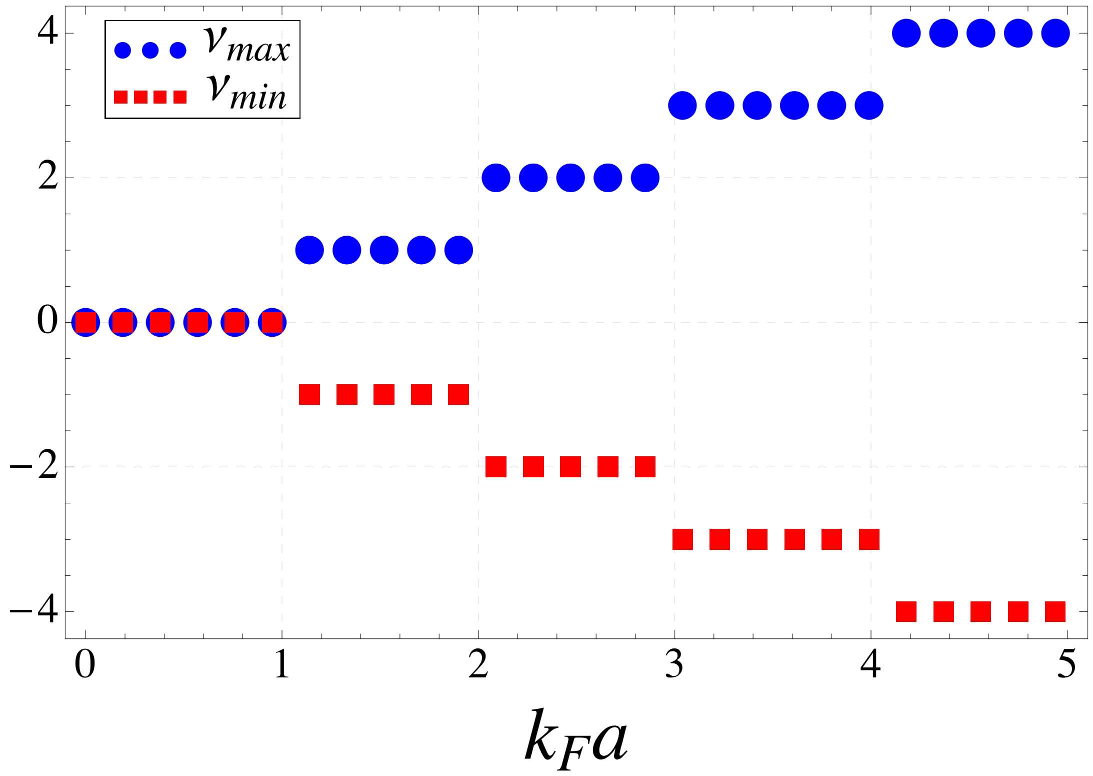
Period $\lambda_a = 1/k_F$

Sommerfeld-Wilson

$$a_\nu (\hbar k_F) = \hbar \nu$$

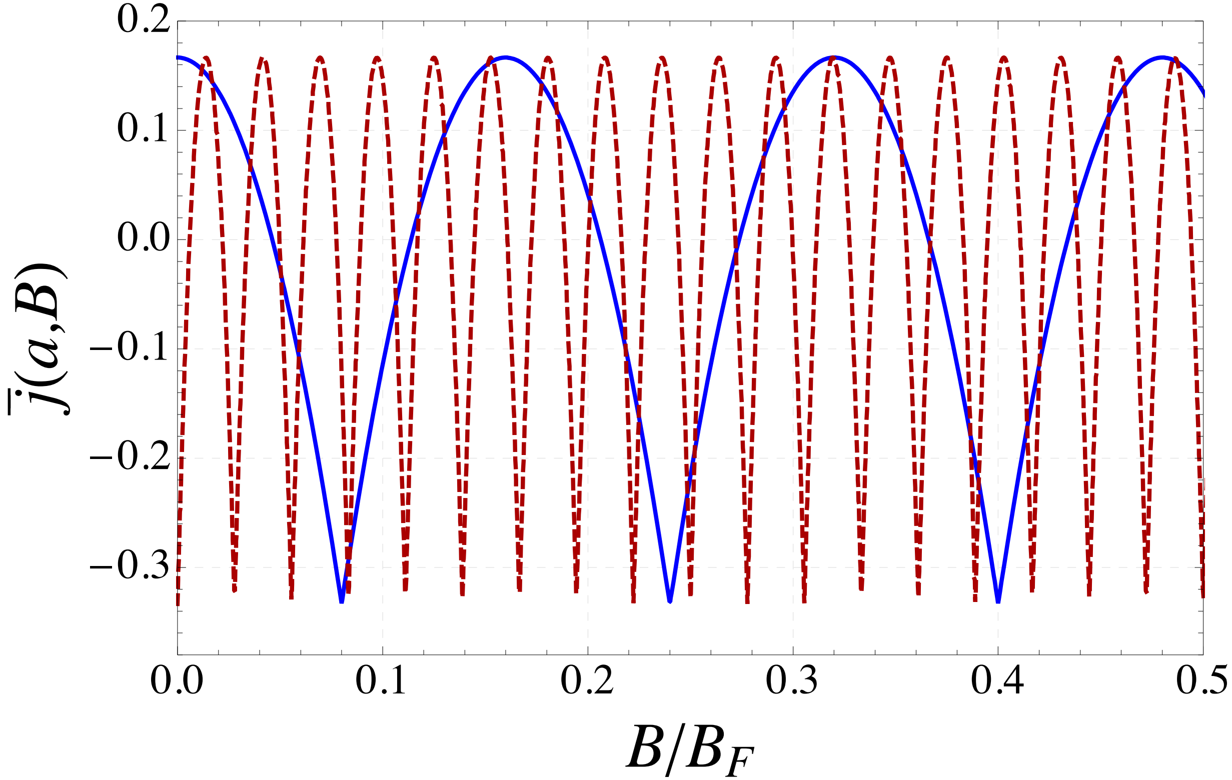
$$\bar{j}\,(a, B=0) = \left(\frac{j_z(a,0) - j_z^c}{j_F}\right)(k_F a)^2$$

Results



B = 0

Results

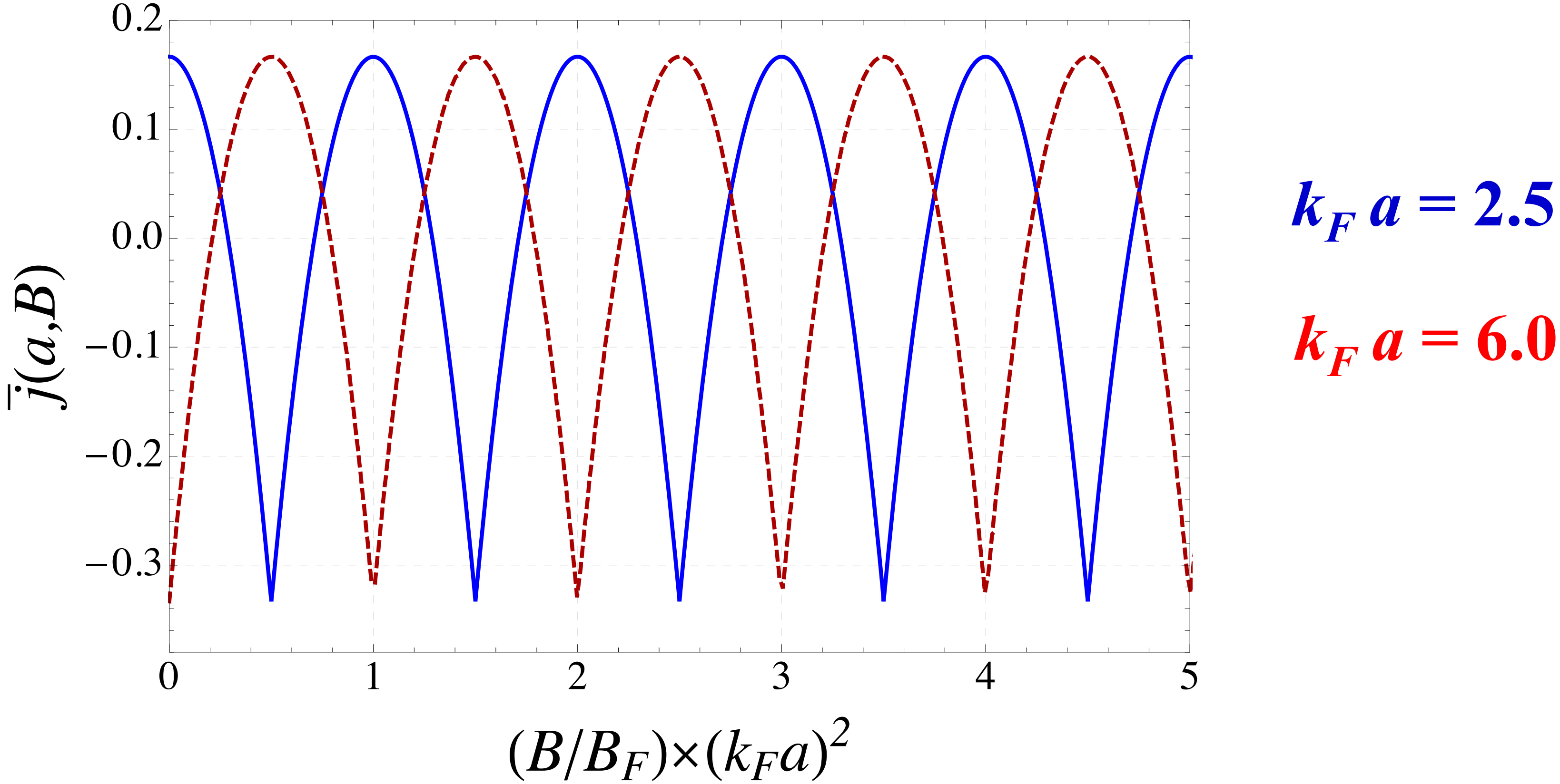


$k_F\, a = 2.5$

$k_F\, a = 6.0$

$$\bar{j}\,(a,B) = \left(\frac{j_z(a,B) - j_z^c}{j_F}\right)(k_F a)^2$$

Results



$$\bar{j}\,(a,B) = \left(\frac{j_z(a,B) - j_z^c}{j_F}\right)(k_F a)^2$$

Results

- **Period of oscillation**

$$(\lambda_B / B_F) \times (k_F a)^2 = 1 \quad \Rightarrow \quad \lambda_B = B_F / (k_F a)^2$$

➔ $$\boxed{\lambda_B = \Phi_B / (\pi a^2)}$$

$\Phi_B =$ **quantum of magnetic flux**

Conclutions

- The current density oscillates in the radial direction about the classical value, both as a function of the radius *a* and the magnetic field B;
- In all cases, the amplitude of oscillations decays as $1/(k_F a)^2$
- Period of oscillations with a ($B = 0$): $\lambda_a = 1/k_F$
- Period of oscillations with B (fixed a): $\lambda_B = \Phi_B/(\pi a^2)$

Happy birthday, Ivan.

Thank you!
Muchas gracias!