ANGULAR DEPENDENT CRITICAL FIELDS IN NbTi-Ge SUPERLATTICES IN THE WEAKLY LOCALIZED REGIME

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ABSTRACT

The angular dependence of the upper critical fields, $H_c(\theta)$ for a set of NbTi-Ge superlattices were studied at various temperatures. The behavior of $H_c(\theta)$ at lower temperatures deviates from the Tinkham expression which is expected to be valid only in the Ginzberg-Landau regime close to T_c. We examine a model for calculating $H_c(\theta)$ involving the lowest eigenvalue of the gauge invariant diffusion equation (subject to boundary conditions appropriate to a slab) in the de Gennes expression for the upper critical field of a dirty superconductor at all temperatures. The disorder related localization and interaction effects as well, as the paramagnetic limiting effect, are also considered.

INTRODUCTION

In a layered superlattice structure, the superconducting behavior is of interest because both the superconducting and barrier layer thicknesses may be made comparable to characteristic length scales describing the superconducting state.

One of the most dramatic demonstrations of these finite layer thickness effects is the dimensional crossover of superconductivity as revealed by the upturn of the parallel critical fields $H_{c2\|}$ (T) at a temperature T* where the effective coherence length perpendicular to the layers, $\xi_\perp(T)$, becomes comparable to the modulation wavelength.[1-11] At temperature below T*, a 2D behavior in $H_{c2\|}$ (T) is observed[7,9,11];

$$H_{c2\|}(T) = \phi_0/2\pi\xi_\|(T)\ (D_s/\sqrt{12}) \quad \propto \quad (1-T/T_c)^{1/2}, \tag{1}$$

where D_s is the superconducting layer thickness and $\xi_\|$ is the in - plane coherence length. In the 2D regime, the angular dependence of H_{c2} at a temperature near T_c is given by the Tinkham expression:

$$\frac{H_{c2}(\theta)}{H_{c2\perp}}\ |\cos\theta| + \frac{H^2_{c2}(\theta)}{H^2_{c2\|}}\ \sin^2\theta = 1\ , \tag{2}$$

where $H_{c2\|}$ is given by Eq. (1), $H_{c2\perp} = \phi_0/2\pi\ \xi^2_\|$; θ is the angle between magnetic field and the film normal. A particular feature of Eq. (1) is the existence of a cusp structure for $\theta = \pi/2$. We note that Eq. (2) is strictly applicable only near the zero field transition temperature, T_c, where Ginzberg-Landau theory is valid. Indeed, we have observed a clear deviation from Eq. (2) when $H_{c2\|}(\theta)$ is studied at temperatures far from T_c, and this deviation will be a focus of this paper.

From Eq. (1), $H_{c2\|}$ (T) in 2D is inversely proportional to D_s; idealy a 2D film with a small D_s would have a significantly enhanced $H_{c2\|}$. However, this enhancement due to decreasing D_s is usually affected by higher disorder in the films and is also subject to paramagnetic limiting.[12] There is now a general concensus that disorder and its related localization and interaction effects reduces T_c due to an enhanced Coulomb repulsive interaction and a depressed density of electron states at the Fermi level.[13,14,15]

A proper theory of a disordered superconducting superlattice, in particular $H_{c2}(\theta)$ at various temperatures, will have to include the effects of disorder and paramagnetic limiting, and a provision for the interlayer coupling; to our knowledge such a theory is not available.

In this paper, we will assume that the Ge layer thickness is sufficiently thick that the Josephson coupling between layers, in the temperature regime studied, can be neglected. Hence, the existing theories for superconducting slabs can be applied to analyze our $H_{c2}(\theta)$ data.

The sample preparation and characterization is reported elsewhere[16].

THEORETICAL MODEL

We limit ourselves to the case of a superconductor/insulator superlattice where the (Josephson) coupling between the superconducting layers is negligible and the system may be regarded as stack of independent superconducting slabs. Near the zero field transition temperature, the Ginzberg-Landau (GL) equation is written as

$$-\frac{\hbar^2}{4m}\left(\nabla - \frac{2ie}{\hbar c}\vec{A}\right)^2 \psi + a\psi = 0, \tag{3}$$

where ψ is the complex "order parameter", and $\vec{A}$ is the vector potential for the external field. For an arbitrary field orientation, the vector potential is written as:

$$\vec{A} = \hat{y}(x\cos\theta - z\sin\theta)\, H; \tag{4}$$

here $\vec{H}$ is in the x,z plane and $\hat{z}$ is the film normal direction. Writing $\psi(\vec{r}) = u(x,z)e^{iky}$, the lowest eigenvalue of the linearized GL equation, with appropriate boundary condition for a slab with thickness D_s, is[10]

$$-a = \frac{1}{2}\hbar\omega_c|\cos\theta| + \frac{m\omega_c^2 D_s^2}{12}\sin^2\theta, \quad \omega_c = \frac{eH}{mc}\,. \tag{5}$$

With the previous definitions for $H_{c2\|}$ and $H_{c2\perp}$, the above equation is equivalent to the Tinkham expression, Eq. (2).

To study the $H_{c2}(\theta)$ at an arbitrary temperature, we use the linearized self-consistent field method as implemented in de Gennes.[17] In the dirty limit, the superconducting transition temperature is determined by a correlation function $g(\vec{r},\vec{r}',t)$ satisfying a modified (gauge invariant) diffusion equation,

$$D[\vec{\nabla} - \frac{2ie}{\hbar c}\vec{A}(\vec{r})]^2 \, g(\vec{r},\vec{r}',t) = \frac{\partial}{\partial t} g(\vec{r},\vec{r}',t) \quad , \tag{6}$$

subject to the condition that g approaches a δ function as $t \to 0$, and $\hat{n}\cdot(\vec{\nabla} - \frac{2ie}{\hbar c}\vec{A})\, g = 0$ at a vacuum-superconductor boundary where $\hat{n}$ is the surface normal . The eigenfunctions, $g_n(\vec{r})$, which satisfy Eq. (6) are then given by

$$D[\vec{\nabla} - \frac{2ie}{\hbar c}\vec{A}(\vec{r})]^2 \, g_n(\vec{r}) = \Omega_n g_n(\vec{r}), \tag{7}$$

and we express $g(\vec{r},\vec{r}',t)$ as

$$g(\vec{r},\vec{r}',t) = \sum_n g_n^*(\vec{r}') \, g_n(\vec{r}) \, e^{-\Omega_n t} \quad . \tag{8}$$

The lowest eigenvalue, Ω_0, of Eq. (8) is substituted in the following expression to obtain the temperature dependence of the upper critical field:

$$\ell n(T/T_{c0}) = \chi(\hbar\Omega_0/2\pi k_B T) = \chi(eDH/2\pi c k_B T), \tag{9}$$

where $\chi(z) = \psi(\frac{1}{2}) - \psi(\frac{1}{2} + z)$; ψ is the digamma function. Note Eq. (7) is identical to Eq. (3) if we identify D as $\hbar^2/4m$ and Ω_0 as a; the boundary condition is also the same. $H_{c2}(\theta)$ is then given by

$$\ell n(T/T_{c0}) = \chi(a'/2\pi k_B T), \tag{10}$$

$$a' = (eD/c)\, H|\cos\theta| + (2e^2 D/\hbar c^2)(D_s^2/12)H^2\sin^2\theta \; . \tag{11}$$

We note that T_{c0} in Eq. (10) is the bulk zero field superconducting transition temperature. In a disordered quasi-2D system, the real T_c will generally be lower than T_{c0} and is given by the Maekawa and Fukuyama[13] equation:

$$\ell n(T_c/T_{c0}) = -(\lambda_1/2)[\ell n(5.4\xi_0 T_{c0}/\ell T_c)]^2 - (\lambda_1/3)[\ell n(5.4\xi_0 T_{c0}/\ell T_c)], \tag{12}$$

where, $\lambda_1 = 1.25 \times 10^{-5} R_\square \, g_1 N(0)$; $R_\square$ is the sheet resistance and $g_1 N(0)$ is the exchange interaction constant; ξ_0 and ℓ are the BCS coherence length and mean free path, respectively.

Disorder also affects the upper critical field and has been studied by Maekawa, Ebisawa and Fukuyama.[14] They obtained the following expressions:

$$\ell n(T/T_{c0}) = \chi(B^*) + R_{HF} + R_V , \tag{13}$$

$$R_{HF} = -\frac{\lambda_1}{2}[\ell n(5.4\frac{\xi_0}{\ell}\frac{T_{c0}}{T_c})]^2 - \lambda_1 \ell n(5.4\frac{\xi_0}{\ell}\frac{T_{c0}}{T_c})\chi(B), \tag{14}$$

$$R_V = -\frac{\lambda_1}{3}[\ell n(5.4\frac{\xi_0}{\ell}\frac{T_{c0}}{T_c})]^3 - \lambda_1[\ell n(5.4\frac{\xi_0}{\ell}\frac{T_{c0}}{T_c})]^2\chi(B), \tag{15}$$

where

$$B^* = eDH[(1-\lambda\ \ell n\ (5.4\frac{\xi_0}{\ell}\frac{T_{c0}}{T_c})]/2\pi k_B cT,\ \lambda = 1.25\times10^{-5}R_{\square} \tag{16}$$

and $B = eDH/2\pi k_B cT$. Here R_{HF} and R_V are the *"self-energy"* and *"vertex"* corrections both of which arise from the Coulomb interaction effect. The factor $(1-\lambda\ \ell n\ 5.4\ \xi_0\ T_{c0}/\ell\ T_c)$ in B^* (Eq. 16) is a correction to the diffusion constant due to the localization effect. We note that in the limit of vanishing disorder, Eq. (13) reduces to the Maki-de Gennes - Werthamer expression Eq. (9); so we are encouraged to use the angular dependence of Eq. (11) in Eqs. (13)-(15) to obtained an expression for $H_{c2}(T,\theta)$ of a disordered, quasi-2D system. Finally, the electron spin paramagnetic contribution is introduced by adding a term $3/2(\tau_{s0}/\hbar)(\mu H)^2/2k_BT$ to the argument B^*.[6,18] This expression will be used to analyze our $H_{c2}(\theta)$ data.

RESULTS AND DISCUSSIONS

The angular dependence of the critical field, $H_{c2}(\theta)$, is shown in Fig. 1 for four NbTi-Ge samples (the sample characteristics are shown in Table 1). The sample with the thickest D_s (221Å) showed isotropic $H_{c2}(\theta)$; hence each NbTi layer is by itself 3D like.
The remaining three samples exhibit a large anisotropy with cusps about the parallel field orientation ($\theta = 90°$) indicating 2D behavior. The solid lines in Fig. 1 are obtained from the Tinkham expression by fixing the two end points at $\theta = 0°$ and $90°$. We see that at higher temperatures, curves E and D, are consistent with the Tinkham expression, while at lower temperature (far from the zero field T_c), curves A, B and C, a considerable deviation from the Tinkham theory is clearly seen.

Identifying eDH/c of Eqs. (13)-(15) with a' of Eq. (11) and including the paramagnetic term, we solved Eq. (13) numerically for $H_{c2}(\theta)$ at fixed temperatures. The two free parameters D and D_s are fitted to the $H_{c2\perp}$ and $H_{c2\parallel}$ data, respectively, so the fitting is essentially a two point fitting. The results are shown in Figs. 2(a), (b) and (c) (model 1). The fitting parameters are shown in Table 2. We see that the fit to the experimental data is improved, but a considerable deviation still exists. If we arbitrarily assume that only the *vertical* component of the external field affects the self-energy and vertex corrections to the upper critical field (ie. we simply replaced H by $H|\cos\theta|$ in Eqs. (14) and (15)), we obtain a more satisfactory fit to the experimental data for the two thinner samples. It is unclear at the present why this second model anzatz fits the $H_{c2}(\theta)$ better. One might postulate that the localization contribution is less significant in the parallel field orientation, due to the confinement of the *"orbital"* motion of the electrons in the z direction[14].

TABLE I Sample characteristics and relevant parameters.
D_s and D_{Ge} are the layer thicknesses of $Nb_{0.53}Ti_{0.47}$ and Ge, respectively.

Sample	D_s(Å)/D_{Ge}(Å)	T_c(K)	$R_\square(\Omega)$
a	59/38	4.78	196
b	96/45	6.5	100
c	144/32	7.5	61
d	221/45	8.9	47

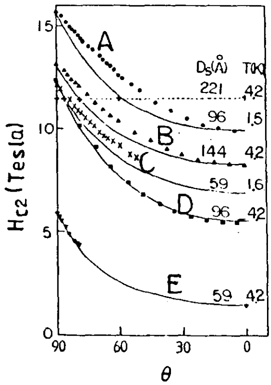
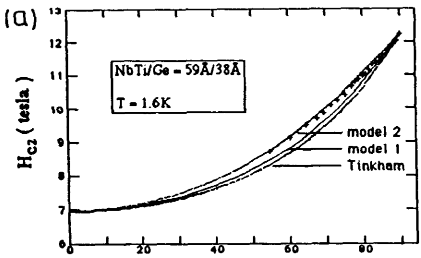


Fig. 1 Angular dependence of H_{c2} fitted with the Tinkham theory. Here the solid lines are from the Tinkham expression and the dotted line is a guide eye. The symbol designations are: ▼, 59Å/38Å (4.2K); x, 59Å/38Å (1.6K); ■, 96Å/45Å (4.2K);
•, 96Å/45Å (1.5K); ▲, 144Å/32Å (4.2K); +, 221Å/45Å (4.2K).

Fig. 2 $H_{c2}(\theta)$ fitted with the de Gennes - MEF expression Eq. (13). Model 1 uses the full expression for a′(Eq. 11), while Model 2 uses only the normal component of a′ in Eqs. (14) and (15); + is the experimental data.



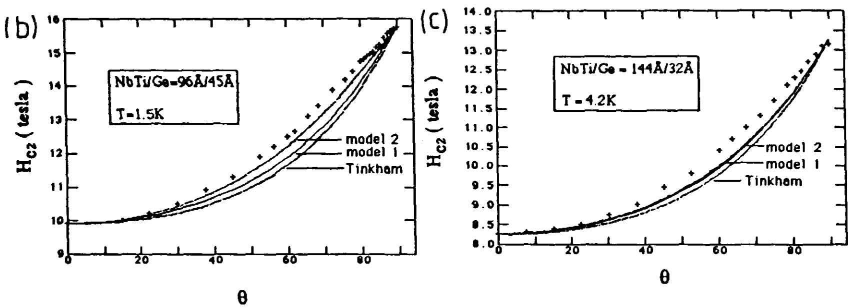


TABLE II Parameters used in the numerical fit.

Sample		Model 1		Model 2	
NbTi(Å)/Ge(Å)	T(K)	D(cm²/s)	D_s(Å)	D(cm²/s)	D_s(Å)
59/38	1.6	0.44	119	0.44	81
96/45	1.5	0.433	113.5	0.434	91.8
144/32	4.2	0.365	125	0.367	117

We note that neither model fully accounts for $H_{c2}(\theta)$ of the sample with D_s = 144 Å. This may be due to an increasing 3D nature of the thickner samples for which the 2D theory (Eq. 13) is no longer valid. We therefore conclude that further refinements of the above model are required. Extensions of the present model to anisotropic, Josephson coupled superlattices is also desirable.

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