Thin-film modeling for mechanical measurements: Should membranes be used or plates?

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The two limits for modeling vibrating thin films, viz as a plate or as a membrane, are discussed. In terms of these limits the sources of experimental uncertanties are identified and the advantages of using films which can be modeled as membranes are explained. The mathematical formalism for treating films as vibrating membranes is developed and relations for self-consistency checks are presented. The drive mechanisms used to generate vibrations in the films are also discussed.

I. INTRODUCTION

The mechanical properties of thin films are often determined from the frequencies of their normal modes of vibration. The frequencies of these modes are most often calculated assuming that the material behaves as a "plate," i.e., that the dominant restoring forces are due to the "bending" of the material.[1] A good example is the "vibrating reed"[2] which models a thin film as a rigid rectangular plate (clamped at one end) and measures its resonant modes to find its flexural modulus. Experimentally, however, the measurement of the film's bending force is complicated. Being proportional to the film thickness, at small thicknesses, it may be dominated by stray forces that are normally present under experimental conditions.

There are two main sources from which stray forces originate: (i) clampings of the film and (ii) deformations such as warpings and wrinkles. Systematic measurements on clamped thin metal disks have shown (at thicknesses below 25 μm) that clamping forces are sufficiently large to considerably shift or alter the resonant modes.[3] Similar measurements have been performed by vibrating reed on (5 μm thick) rectangular Cu/Ni films and have revealed that deformations can influence the resonant modes by 200%–300%.[4] These facts (as well as some other controversial results[1,4–6]) have raised doubts whether techniques based on plate modeling produce reliable results.

When the dominant restoring force of a film is the tension applied along its boundary it will behave as a "membrane." In a recent article we described how an elastic property could be measured by forming a film into a membrane. Here we present detailed formal investigations which show a number of advantages in using membrane techniques when dealing with thin film samples. Most importantly we find the effects of stray forces can easily be eliminated.

Below we will discuss the conditions that must be met in order for a circular film clamped at its outer boundary to be treated as a plate or as a membrane. Expressions for the errors introduced when treating a film as a membrane (plate) will be discussed. The roles of plate rigidity and tensions applied during the mounting of the sample will be evaluated to determine which approximation is more appropriate.

II. BASIC FORMALISM

We model a thin film as an elastically isotropic circular disk under a uniform biaxial tension T. Ignoring the effects of shear (tension), the expression for its vibrational energy is written as

$$E_{\text{vib}} = U_{\text{plate}} + U_{\text{membrane}} + \text{KE}. \tag{1}$$

U_{plate} is the plate bending energy, U_{membrane} is the film's restoring potential energy which comes from its tension, and KE is the kinetic energy. Taking the plane of the film in the x-y plane, and writing in polar coordinates the integral expressions for each term,[7] we have

$$E_{\text{vib}} = \frac{Eh^3}{24(1-\eta^2)} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 \zeta}{\partial^2 r} + \frac{1}{r}\frac{\partial \zeta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \zeta}{\partial \theta^2} \right)^2 r\, d\theta\, dr + \frac{Th}{2} \int_0^{2\pi} \int_0^a \left[\left(\frac{\partial \zeta}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial \zeta}{\partial \theta} \right)^2 \right] r\, dr\, d\theta + \rho \frac{h}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial \zeta}{\partial t} \right)^2 r\, dr\, d\theta, \tag{2}$$

where ζ is displacement perpendicular to the film's plane, t is time, η is the Poisson's ratio, E is the Young's modulus, h is the film thickness, a is the radius, T is tension (force per unit area), and ρ is the density. This equation can be simplified by variational minimization to yield the differential equation of the film's motion;

$$\frac{E h^2}{12(1-\eta^2)} \nabla^4 \zeta - T \nabla^2 \zeta + \rho \frac{\partial^2 \zeta}{\partial t^2} = 0. \tag{3}$$

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For simple harmonic motion $\zeta = \xi e^{i\omega t}$, where ω is the angular frequency. In terms of the Laplacian ∇^2, Eq. (3) now represents a second-order polynomial and can be written as

$$(\nabla^2 + \alpha^2)(\nabla^2 - \beta^2)\xi = 0. \quad (4)$$

The expressions for α^2 and β^2 are

$$\alpha^2, \beta^2 = -[\pm T - \sqrt{T^2 + (\rho\omega^2 h^2 E)/3(1-\eta^2)}]/[Eh^2/6(1-\eta^2)]. \quad (5)$$

The solutions to Laplace's equation are well known and the most general solution for ξ (in polar coordinates) is given below[8]

$$\xi(r,\theta) = {\cos \atop \sin}(m\theta)[AJ_m(\alpha r) + BI_m(\beta r)]. \quad (6)$$

J_m represents an mth order Bessel function and I_m is its hyperbolic counterpart whose argument βr is an imaginary variable. A and B are constants to be determined from the boundary conditions.

In the case of large T's, membrane properties will dominate and the problem can be simplified by noting that $\beta \sim 0$. Using I_m's polynomial expansion, this implies that $I_m(\beta r) \sim 0$, and since we are dealing with a film whose edges are clamped at $r = a$ [i.e., $\xi(a,\theta) = 0$], we have $J_m(\alpha a) \sim 0$. This condition requires the values of αa to be equal to the roots of J_m. After some algebra the expression for the film's fundamental mode is given by

$$\nu_{01} = 0.3826 \left(\frac{T}{\rho a^2} + \frac{0.4823Eh^2}{\rho a^4(1-\eta^2)}\right)^{1/2}, \quad (7)$$

where ν_{01} is the frequency of the fundamental mode, and the subscripts, respectively, denote the number of the nodal lines, and circles. Setting $E = 0$, this expression correctly reduces to that of an ideal membrane with no plate rigidity.

Another method to find the film's fundamental mode is by approximate means. For this purpose we use the Rayleigh–Ritz method and we choose $J_0(\beta r)$ as the trial function—$J_0(\beta r)$ is the exact solution when $E = 0$. The integrals involving Bessel functions were calculated by using the computer program Mathematica (version 1.1).[9] The result of Rayleigh–Ritz approximation is equal to the result obtained using Eq. (7)—to the third decimal place.

When T is small the film's plate rigidity will dominate its restoring force. In the strictest mathematical sense the disk will behave as a rigid plate only when $T = 0$. Experimentally, however, this condition need not be fulfilled and the film can still be considered as a plate as long as T is small. The degree to which T affects the vibrational modes can be examined by using the Rayleigh–Ritz method. For the trial function we use the wave function of the fundamental mode of a rigid circular plate with no tension at its boundary. Its functional form is given by[8]

$$\xi(r,\theta) \propto [I_0(\gamma a)J_0(\gamma r) - J_0(\gamma a)I_0(\gamma r)], \quad (8)$$

where γ is a constant equal to $1.015\pi/a$. By using Mathematica we evaluate the integrals and find that resonant frequency is given by

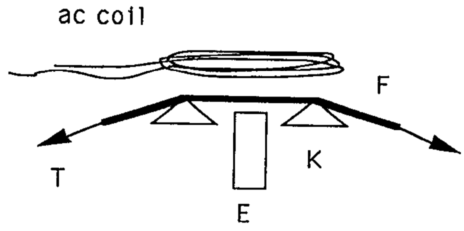

FIG. 1. Schematic diagram of the vibrating membrane device. F is the film, T is tension, K is the circular knife-edge support, and E is the receiver electrode.

$$\nu_{01}^p = 1.0476 \left(\frac{T}{\rho a^2} + \frac{0.2088Eh^2}{\rho a^4(1-\eta^2)}\right)^{1/2}. \quad (9)$$

From Eq. (9) it can be seen that for typical values [of $E/(1-\eta^2) = 10^9$ Pa, $h = 10$ μm, $a = 0.005$ m] the tension must satisfy $T \ll 10^3$ Pa in order for a film to be treated as a plate. Tensions this low are almost impossible to achieve since they are comparable to the tension produced by the films own weight. If in the parameters given above one requires the tension affect the resonant frequencies of less than 1%, T must be less than 20 Pa. We will therefore now focus, in more detail, on membranes and discuss the advantages of using them.

Returning to Eq. (7) and the estimate given above, it can be seen that it is relativelly simple to apply tensions which are sufficiently large to guarantee that the films behave as membranes and that their normal modes are given by Eq. (7) with negligible contribution from their plate rigidity.

The schematic of the setup for membrane measurements is shown in Fig. 1. We have used this setup for biaxial modulus measurements of metallic[3,10] and polymer films.[11] As shown in the figure the thin film is tensioned over a circular knife-edge support. An electrode positioned beneath the film detects its vibrations (induced by a coil) and its strain is measured using an optical microscope. In this design the clamping forces are uniformly produced by the film's own biaxial tension, as it is stretched over the fixed knife-edge support. By this means (since the film is held under its own isotropic biaxial tension) no unknown clamping forces are introduced to interfere with the results of the measurements.

Another advantage in using a membrane is that by using large tensions deformations are easily removed. Their removal eliminates their stress fields, or at least minimizes their influence on resonant modes. In this respect a membrane is superior to a plate because its flexible behavior allows removal of deformations and unlike a plate it is not constrained to preserve the thin-film's rigid behavior which in practice is difficult to achieve when large stray forces are present.

These advantages have led to development of membrane techniques for metallic,[3,10] polymer,[11] and semiconducting[12,13] materials. Since these techniques do not use plate modeling they are free from problems due to stray

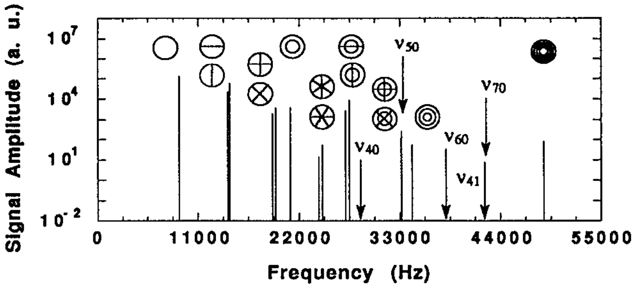

FIG. 2. Signal amplitude vs the square of frequency for a $\sim 15.5\ \mu$m aluminum film. The configurational representation of each mode is shown above its resonant peak. The arrows point at missing modes. These modes are labeled with ν_{ij}'s; i and j are, respectively, the number of nodal circles and lines.

forces and are thus capable of producing more reliable results.

III. TENSION ANISOTROPY AND SPLITTING OF RESONANT MODES

An anisotropic tension will cause the splitting of degenerate modes and the degree to which the modes are split is a measure of anisotropy. The anisotropy either comes from the film's own internal stresses or clamping forces at its boundaries. Here we will discuss the latter problem which comes from tensions to the film during its mounting. We will use the Rayleigh–Ritz approximation to find expressions for the frequencies of the first and second resonant modes of a circular membrane. To simplify our calculations we assume $E=0$. Since E is an isotropic parameter this choice of E will not change the symmetry of the problem and thus affect the splitting of the modes.

The effects of anisotropy are seen in Fig. 2 which shows the resonant modes of a ~ 15.5-μm-thick Al film. The second mode is doubly degenerate and its splitting is ~ 100 Hz. The splitting represents a $\sim 1\%$ deviation from the expected value for this mode.

We assume that the total tension applied to the film can be written as

$$T_{\text{Total}} = T + \epsilon f(\theta), \tag{10}$$

where T is tension explicity applied during the experiment and which is independent of θ by design and $\epsilon f(\theta)$ is the tension applied during mounting and which could be direction dependent. $f(\theta)$ is an arbitrary function of θ, and ϵ is a tension that is assumed to be very small compared to T. The approximate solutions for the first (fundamental) and second modes are, respectively, shown below:

$$\nu_{01} = 0.3827\sqrt{\frac{T}{a^2\rho}}\left(1+\frac{\epsilon}{2T}\langle f(\theta)\rangle\right), \tag{11}$$

$$\nu_{11} = 0.6098\sqrt{\frac{T}{a^2\rho}}\left\{\left(1+\frac{\epsilon}{2T}\langle f(\theta)\rangle\right) \pm\left(0.148\,17\frac{\epsilon}{T}\right)\sqrt{p^2+q^2}\right\}, \tag{12}$$

where $\langle f(\theta)\rangle$ is the average of $f(\theta)$ and defined as

$$\langle f(\theta)\rangle = \frac{1}{2\pi}\int_0^{2\pi} f(\theta)d\theta, \tag{13}$$

and similarly,

$$p = \langle f(\theta)\cos 2\theta\rangle, \tag{14}$$

$$q = \langle f(\theta)\sin 2\theta\rangle. \tag{15}$$

In Eq. (11) the first term represents the unperturbed portion of ν_{01} and the second term gives the shift due to tension's anisotropy. Equation (12) shows that ν_{11} is split into two parts. These parts are symmetrically positioned about a central frequency which itself is shifted with respect to ν_{11}'s unperturbed portion (the first term). Equation (12) also shows that the splitting should decrease as tension is increased. Experimentally we do find that the splitting of the second mode decreases as T is increased; it does not however decrease as rapidly as predicted by Eq. (12). We believe that this could be due to other sources of film anisotropy such as thickness variations or to our assumption that ϵ and/or $f(\theta)$ in Eq. (10) are also weakly dependent on the tension T.

We have verified that the major contribution to the splitting of the second mode is due to anisotropic tensions during mounting since in all cases where splittings larger than 2% were observed, they could be greatly reduced by remounting the sample. In all our measurements the self-consistency of ν_{01} and ν_{11}, and the splitting of the latter are an integral part of the experiment and are routinely carried out each time a new sample is mounted.

The influence of a membrane's rigidity in the presence of an isotropic tension is of interest where thick films such as polymer samples are studied. Using the approximation techniques described above and the same trial functions, we find that the first and second vibrational modes are given by

$$\nu'_{01} = 0.3827\sqrt{\frac{T+\Omega}{a^2\rho}}\left(1+\frac{\epsilon}{2(T+\Omega)}\langle f(\theta)\rangle\right), \tag{16}$$

$$\nu'_{11} = 0.6098\sqrt{\frac{T+\Delta}{a^2\rho}}\left\{\left(1+\frac{\epsilon}{2(T+\Delta)}\langle f(\theta)\rangle\right) \pm\left(0.14817\frac{\epsilon}{T+\Delta}\right)\sqrt{p^2+q^2}\right\}, \tag{17}$$

where $\Omega = 25.4969D$ and $\Delta = 64.7314D$. Here, D is defined as $Eh^2/12a^2(1-\eta)^2$. The rest of the parameters are the same as already described above. An interesting feature, as expected, is that the sample's rigidity, being isotropic, only adds to tension and by itself does not change the functional forms of the expressions of the resonant modes.

IV. DRIVE MECHANISM

Although the capacitive system used to detect the resonant frequencies was described in Ref. 10, it was only briefly mentioned that the vibrations could be induced by

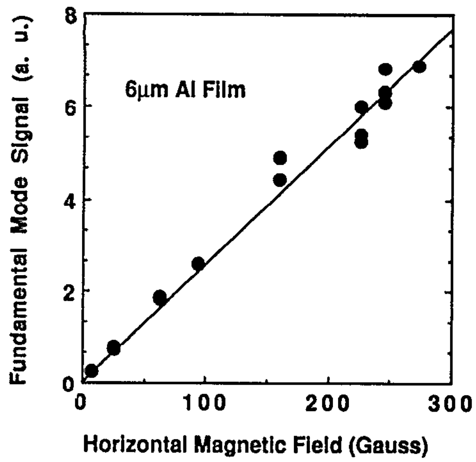

FIG. 3. Signal amplitude vs horizontal B field intensity for a 6 μm aluminum film.

either an electrostatic interaction or by a coil. The former is intuitively simple and requires no further description. The second technique which is less obvious but which has a number of advantages is briefly described in this section.

The force exerted by a coil above a conducting plane has been the subject of numerous articles.[14] The rigorous method for calculating this force has been given by Dodd and Deeds.[15,16] For the case of a vibrating membrane the theory can be simplified in terms of Lorentz interactions of eddy currents and magnetic fields. The magnetic fields are considered to be either induced by the coil or imposed by external means. Disregarding the geometrical terms, and assuming axial symmetry, for each case we write the vertical component of the force as follows:

$$F_z = (J_\phi e^{i\omega t})(B_r e^{i\omega t}), \tag{18}$$

$$F_z = (J_\phi e^{i\omega t})(B_{\text{static}}). \tag{19}$$

In these equations the eddy current is assumed to have only azimuthal component J_ϕ, and the radial horizontal component of the field induced by the coil is denoted by B_r. The coil's angular frequency is ω. B_{static} is a dc field, and is imposed parallel to the plane of the conducting film. Equation (18) is the product of two dynamic terms and represents a force whose frequency, in absence of static fields, is twice the frequency of the coil. By surrounding the membrane by a mu-metal shield we have verified that resonances do indeed only occur at 2ω. Equation (19) shows a different behavior when a static field is involved. Since this field has no time dependence the resonant frequency occurs at ω. For a field as small as the earth's, the signal amplitude is sufficiently large to be easily detected. To investigate the effects of much larger fields an electromagnet was placed with its field parallel to the membrane's plane. Figure 3 shows the signal amplitude versus the magnetic field intensity. A linear behavior is found consistent with the prediction of Eq. (19). The use of dc field has the advantage that large amplitudes can be generated to detect higher harmonics of the membrane. Figure 2 is a typical plot obtained by applying a field of 100 G.

V. CONCLUSION

The membrane technique measures the elastic modulus of a film by finding its stress/strain relationship. Stress is measured dynamically from the resonant modes, whereas strain is found by using a static method. Its advantage over other techniques (such as the vibrating reed) is that the membrane technique is not based on plate modeling. The problem with the plate modeling is that it is valid only for thick films. The thin ones are easily disturbed by stray forces and, to the degree that stray forces cannot be eliminated, the validity of the plate modeling cannot be ensured. In the membrane technique, problems posed by stray forces (from the clampings and deformations) are easily resolved by application of large tensions. The clamping forces cause no problems because the membrane is held in place under its own tension. For this reason stress measurements are free from outside interference, since no external forces are used. Moreover, since large tensions are applied deformations are removed, and their influence (typically of the order of the film's plate bending force) if they still persist can usually be ignored as being too small compared to the tension's restoring force. The membrane technique also offers another advantage in that it requires no knowledge of film thickness. This is very useful when films are studied whose exact thicknesses are not known.

In this article the basic formalism for the vibrational modes of a circular film was developed and its limiting behavior as a plate or as a membrane was discussed. The advantages in using a membrane were explained and it was shown that the behavior of a membrane can be self-consistently examined against the frequency of its resonant modes. Also the shift in a membrane's fundamental mode due to its plate rigidity was found and perturbations caused by anisotropies in tension were derived for its first two modes by approximate means. Moreover, the theory behind the driving mechanism was explained and experimentally tested, and finally utilized to excite the higher harmonics of a vibrating membrane.

ACKNOWLEDGMENTS

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