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An ac biased superconducting flux transformer

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Résumé. — On prévoit un comportement de type Josephson pour le mouvement couplé de deux réseaux de vortex dans deux films minces superposés (le primaire et le secondaire), isolés électriquement et plongés dans un champ magnétique perpendiculaire. Lorsqu'un courant traverse le film primaire, la différence entre les tensions moyennes aux bornes du primaire et du secondaire dues au mouvement des vortex en régime de « flux flow » obéit à une équation identique à celle du modèle d'un lien faible Josephson. Si un courant alternatif est superposé au courant continu, des marches de type Shapiro sont prédites dans la caractéristique courant-tension. Contrairement à l'effet Josephson classique, la position en tension de ces marches dépend du champ magnétique appliqué.

Abstract. — The coupled motion of the vortex lattices of two electrically insulated, superimposed, superconducting thin films (primary and secondary) in a perpendicular magnetic field, is predicted to exhibit a Josephson-like behavior. For a dc applied current flowing in the primary film, the voltage difference between the time-averaged primary and secondary flux flow voltages obeys the equations of a resistively shunted junction model. If an ac current is superimposed on the dc current, Shapiro-type steps are predicted in the current-voltage characteristic. In contrast to the conventional Josephson effect, the voltage positions of these steps can be tuned by the applied magnetic field.

The sensitivity to rf or microwaves of a conventional *Josephson coupled* junction is related to the phase coherence provided by the weak overlap of the superconducting wave function of the two superconducting electrodes forming the device. The effect of an external electromagnetic radiation, of frequency ν, on a Josephson junction is characterized by current steps in the current-voltage characteristics, $I(V)$. The voltages V_n, at which the nth step occurs, are directly proportional to the applied frequency, $V_n = n(h/2e)\nu$ while the ratio $\nu/V_1 = 484\ \mathrm{MHz}/\mu V$ is constant.

A different type of weak coupling can be obtained between two electrically isolated, superimposed, superconducting films in a perpendicular magnetic field. Figure 1 shows the configuration of the two superconducting films separated by a dielectric layer D. The applied

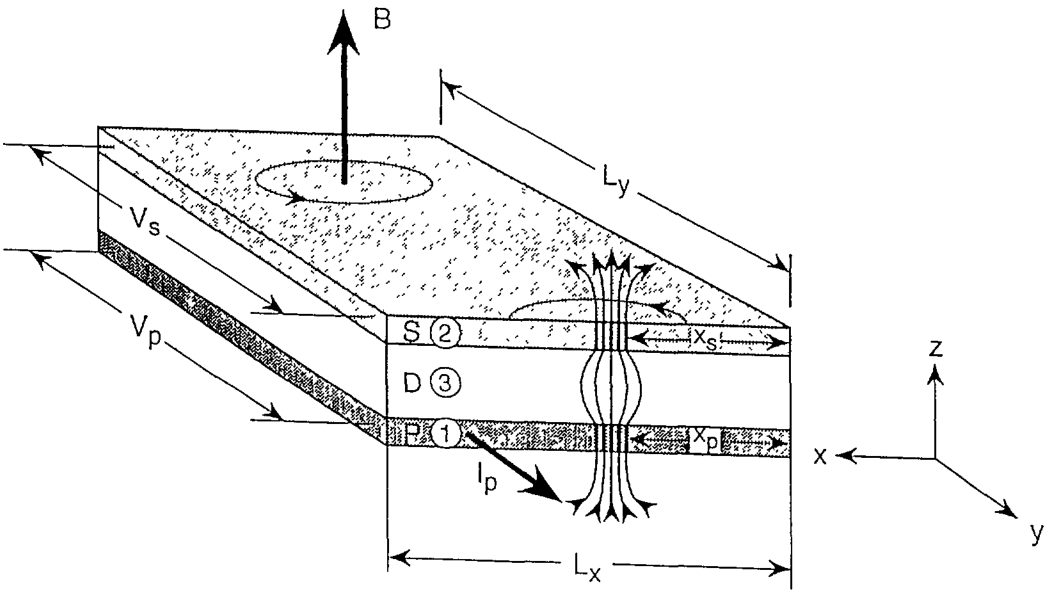

Fig. 1. — Configuration of the d.c. flux transformer : P is the primary film ; D the dielectric layer and S the secondary film.

magnetic field, perpendicular to the films, creates vortex arrays in both films. Quantized vortices of flux $\Phi_0 = h/2e = 2.07 \times 10^{-15}$ Tm^2 form a triangular lattice in each film. In this configuration of the so called « dc flux transformer » [1-5], there exists a *magnetic coupling* between vortices confined to the two superconducting films. When a dc current of sufficient magnitude to unpin the vortices is passed through one of the films (primary P), the vortex lattice is set in motion, inducing a dc voltage along its length. If the magnetic coupling with the superimposed secondary (S) vortex lattice is strong enough, this will be dragged along by the motion of the (P) vortex lattice and a dc voltage appears across the (S) film. By measuring the dc voltage in (S) *versus* the applied dc bias current through (P), it is possible to determine the degree of coupling between the vortex lattices in the two superconducting films. For low bias current, the magnetic coupling is strong and both layers show the same dc voltages corresponding to the coupled motion of vortices in the two layers (Fig. 2a). For very high bias current, the fast motion of the vortex array in the P layer becomes independent of the slow motion of the vortex array in the S layer. This regime is characterized by a finite dc voltage generated in the P layer and a very low voltage in the S layer (Fig. 2c). Therefore, by tuning the dc bias current in the P layer it is possible to induce a continuous cross-over from a fully coupled to a completely decoupled independent motion of vortices of the two layers (Fig. 2b). The coupled motion of vortices implies the existence of a certain phase coherence for the superconducting parameter describing the condensate in the two layers. This coherence is lost when the motion of the vortices in the two layers becomes decoupled (at high dc bias current). In all the previous studies on the dc flux transformer only the dc current in the P or S films has been considered.

In this paper, we predict that a new Josephson-like effect occurs when an additionnal ac current is superimposed at intermediate dc bias currents. We show that this type of magnetic coupling leads to phase coherence and obeys equations analogous to the Josephson effect. If an electromagnetic wave is applied, Shapiro steps appear at voltages V_n which opens up a new

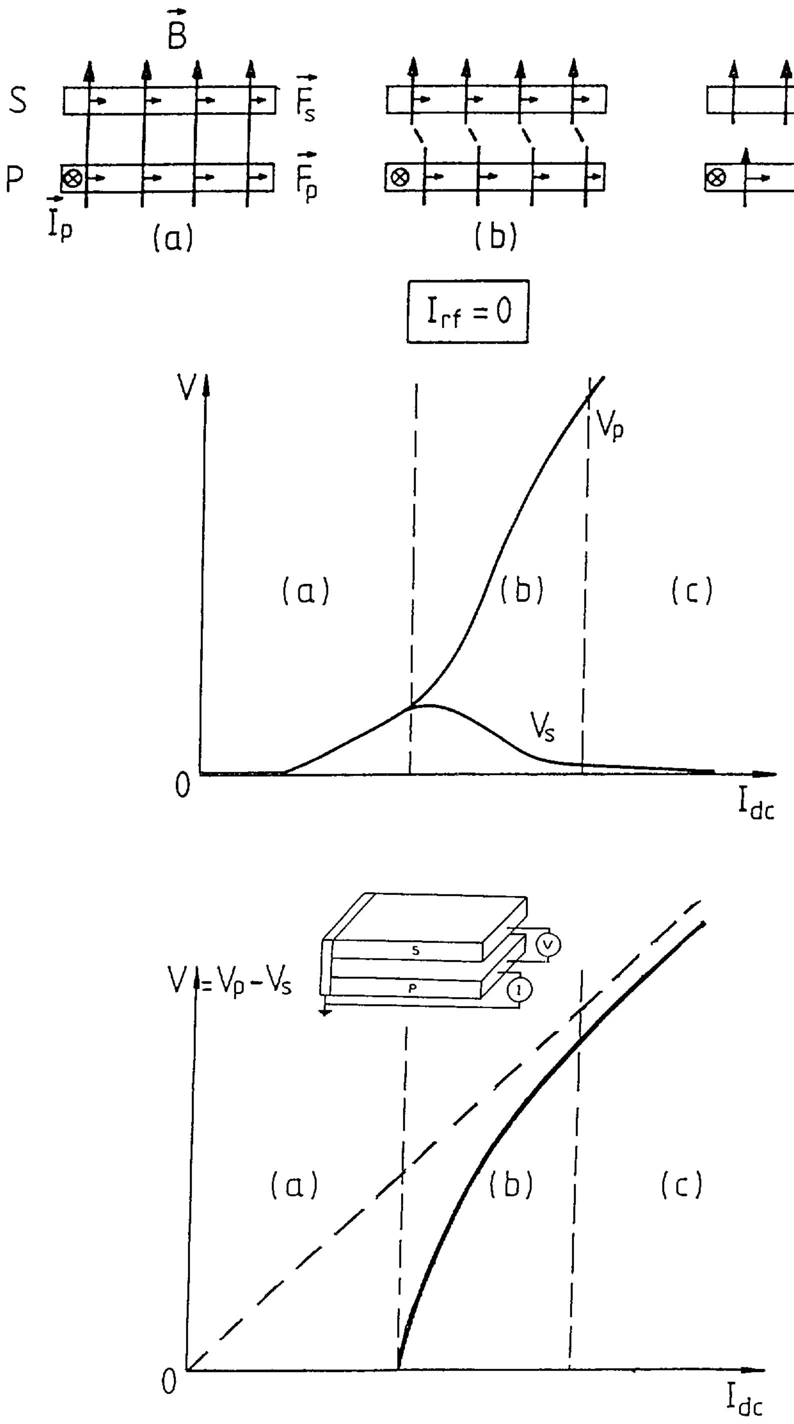

Fig. 2. — Schematics of the motion of the vortices lattices in the primary P and in the secondary S superconducting films. a) Coupled motion of vortices in the two layers with a low bias current I_p in P. b) Weakly coupled regime. c) Decoupled motion of vortices with a high bias current in P. The corresponding current-measured voltages are also shown.

possibility for rf detection devices. These voltages V_n are proportional to the applied frequency, but in contrast to the conventional ac Josephson effect, the constant of proportionality ν/V_1 (and therefore the voltage positions V_n) can be tuned by the applied perpendicular magnetic field.

The models [2-4] describing the motion of the primary and secondary vortices take into account the Lorentz force due to the applied current I_p in the P layer, the pinning force for the individual flux lattices, the viscous drag force proportional to the vortex array speed dx/dt and a periodic coupling force between the two arrays. The coupling force experienced by P is equal and opposite to that felt by S.

In the general case of a current flowing in the primary (I_p) and in the secondary (I_s), the differential equations describing the coupled motion of the primary and secondary vortices are [4] :

$$V_p/R = L_x(\mathrm{d}x_p/\mathrm{d}t)B/R = (I_p - I_c) - I_0 \sin[(2\pi/a)(x_p - x_s)] \tag{1}$$

$$V_s/R = L_y(\mathrm{d}x_s/\mathrm{d}t)B/R = (I_s - I_c) + I_0 \sin[(2\pi/a)(x_p - x_s)] \tag{2}$$

$a = [2\,\Phi_0/((3)^{1/2} B)]^{1/2}$ is the spacing beetween vortices which are located at x_p and x_s as shown in the figure 1. L_x and L_y are respectively the width and the length of the films. For simplicity, the film critical currents and flux flow resistances are assumed to be the same i.e. $I_{pc} = I_{sc} = I_c$ and $R_p = R_s = R$. V_p (and V_s) are the measured voltages across the length L_y. The first term on the right hand side describes the flux flow regime when a current I_p (or I_s) is driven in the P (or S) film. The second term describes the magnetic coupling by assuming for simplicity a sinusoidal coupling force [2]. This force is due to a periodic matching of the two flux lattices in the S and P superconducting layers. I_0 is related to the maximum magnetic coupling force F_m by the relation [4] $F_m = I_0\,\Phi_0/L_x$ and is a function of the insulator thickness, the superconducting film thicknesses and their temperature and magnetic field dependent penetration depths [5-6].

By defining a phase difference

$$\phi = (2\pi/a)(x_p - x_s) \tag{3}$$

and a voltage difference

$$V = V_p - V_s = L_y B d(x_p - x_s)/\mathrm{d}t \tag{4}$$

and subtracting (2) from (1) :

$$V/R + 2\,I_0 \sin\phi = I_p - I_s = I \tag{5}$$

with

$$V = L_x B(a/2\pi)\,\mathrm{d}\phi/\mathrm{d}t = (L_x/2\pi)[2\,B\Phi_0/(3)^{1/2}]^{1/2}\,\mathrm{d}\phi/\mathrm{d}t\,. \tag{6}$$

Equation (5) is formally identical to the equations of the Resistively Shunted Junction Model (RSJM) of a Josephson weak link without capacitance [7] :

$$V/R + I_m \sin\phi = I \tag{7}$$

where $I_m = 2\,I_0$ is the maximum Josephson current and I the current flowing in the weak link. However, the proportionality coefficient between V and $\mathrm{d}\phi/\mathrm{d}t$ (relation (6)) is different from that of a Josephson junction. Thus, the coupled motion of the vortex lattices is analogous to the behaviour of a Josephson junction. For a Josephson junction, when two superconductors are weakly coupled, the phases of their order parameters are correlated. For a superconducting transformer, the motion of the vortices of the two films are correlated. Since the phase $(2\pi/a)(x_p - x_s)$ corresponds to that of a Josephson junction and the relation $V = L_v B(a/2\pi)\, d\phi/dt$ is similar to the Josephson equation $V = (h/2e)\, d\phi/dt$, both a dc and an ac Josephson effects should be present.

For the dc effect, the current-time averaged voltage $I(V)$ characteristics of a Josephson junction in the RSJ model can be obtained by solving equation (5). The solution is [7] :

$$V = 0 \quad \text{for} \quad I < I_m \quad \text{and} \quad V = R(I^2 - I_m^2)^{1/2} \quad \text{for} \quad I > I_m \tag{8}$$

with $I_m = 2 I_0$ for the d.c. transformer [4].

For $I < 2 I_0$, the two superimposed vortex lattices are strongly coupled and locked together. Therefore, the voltages V_p and V_s are identical and given by :

$$V_p = V_s = (R/2)[I_p + I_s - 2 I_c] . \tag{9}$$

This is analogous to the phase locked coupling of a Josephson weak link in the non dissipative state.

For $I > 2 I_0$, the vortex arrays in the P and S layers are weakly coupled. A partial decoupling of the two vortex lattices occurs and one lattice begins to slip relative to the other with the phase difference $(2\pi/a)(x_p - x_s)$. The voltages V_p and V_s are different and a finite voltage V is induced.

In order to verify equation (8) we analysed the experimental results obtained at $T = 1.4$ K by Ekin *et al.* [3] on an Al/SiO/Al flux transformer. As shown in figure (3), the square of the voltage difference V is indeed proportional to the square of the driven current in (P) at different applied magnetic fields. The applied magnetic fields are $B = 4$; 5.9 and 7.9 Gauss and the corresponding decoupling currents $2 I_0$ are respectively 1.8 ; 1.3 and 1 mA.

For the ac effect, an ac current I_{rf} superimposed on the dc current I_{dc} gives rise to a novel Josephson-type effect. Inserting the total current in equation (7) we obtain :

$$V/R + I_m \sin \phi = I_{dc} + I_{rf} \sin 2\pi\nu t . \tag{10}$$

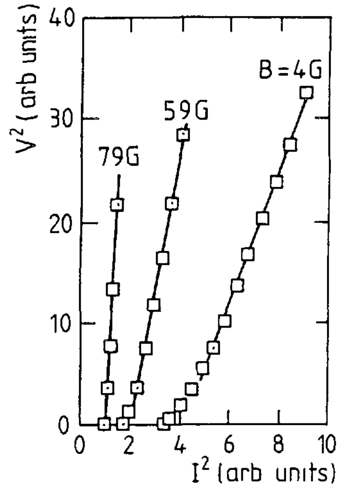

Fig. 3. — Square of the measured voltage difference $V = V_p - V_s$ *versus* the square of the intensity of the primary dc current adopted from Al/SiO/Al flux transformer data [3].

In this case, and in analogy with a Josephson junction, Shapiro steps [8] should appear on the $I(V)$ characteristics shown figure 4. These steps correspond to the average voltages $V_n = nV_1$ where n is an integer and V_1, the first step, is given by equation (6).

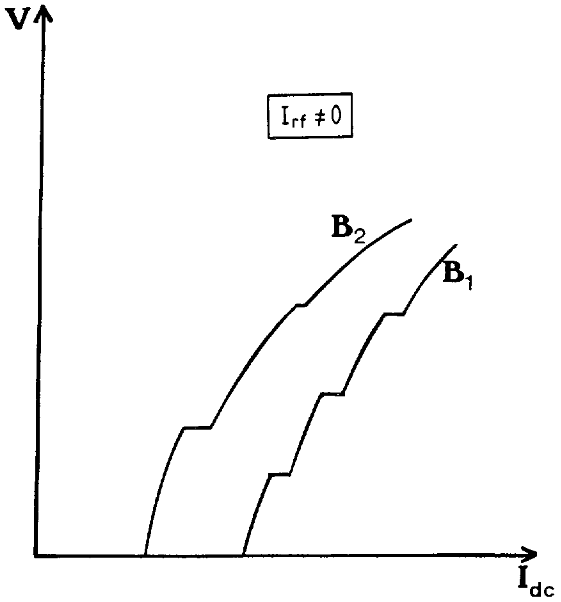

Fig. 4. — Shapiro steps of the ac bias flux transformer for two values of the applied magnetic field ($B_2 > B_1$).

The equivalent frequency to voltage ratio is given by :

$$\nu/V_1 = (aL_y B)^{-1} = (3)^{1/4}[L_y]^{-1}[B \,.\, 2\,\phi_0]^{-1/2} = 20.8\,L_y^{-1}\,B^{-1/2}\,\mathrm{MHz.V^{-1}} \qquad (11)$$

with L_y in meters and B in Tesla. Note that the ratio ν/V_1 is proportional to $B^{-1/2}$ and can therefore be tuned by the applied magnetic field. Figure 4 shows schematically how the voltage positions of the steps are moving for two values of B. For $B = 10^{-3}$ T and $L_y = 10^{-3}$ m, the ratio $\nu/V_1 = 0.658$ MHz/μV. The first Shapiro step will be observed at $V_1 = 10$ μV for an ac current with a frequency of 6.58 MHz.

For an Al/SiO/Al flux transformer [3], the maximum voltage of the weak coupled regime is estimated to be of the order of 0.2 mV for $B = 5.9 \times 10^{-4}$ Tesla. In order to observe several steps in the $I(V)$ characteristic, the frequency of the ac current should be of the order of 10 MHz. As in the case of the Josephson effect, an estimate of the step width is very difficult.

We should point out that an ac quantum interference effect was observed in the flux flow regime of type II superconducting films with a periodically modulated thickness of the order of a micron [9]. Several steps in the $I(V)$ characteristics of Al films appear at well-defined voltages, when the vortex motion is induced by superimposed dc and 10 MHz rf currents in a magnetic field of a few Gauss. These steps appear as a result of the vortex lattice matching to the periodic pinning structure produced by the thickness modulation. In the flux transformer, the vortex array in the secondary film plays the role of the periodic pinning structure. The important advantage of the flux transformer is however that it offers the possibility to tune the ratio ν/V_1 by the magnetic field which changes the pinning structure.

In summary, we have shown that the equations describing the motion of the vortices in a superconducting flux transformer are equivalent to the equations of a Josephson weak link (the Resistively Shunted Junction model). A direct analogy exists between the coupling of the phase of the order parameter in a Josephson junction and the magnetic coupling of vortices in a dc superconducting transformer. A new type of ac Josephson-type effect is predicted and the position of the Shapiro steps is dependent on the magnetic field applied perpendicular to the films. This flux transformer can be used as an rf detector or a calibrated detector for small magnetic fields. A practical advantage might be that the flux transformer is easier to fabricate than a conventional Josephson junction.

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