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Spin-wave theory of exchange-induced anisotropy

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It is shown that exchange interactions of spins across the boundary between ferromagnetic and antiferromagnetic layers can cause a shift in the observed hysteresis loop of the ferromagnetic layer. The effect may be interpreted as a self-energy shift of each ferromagnetic spin due to emission and reabsorption of virtual antiferromagnetic spin waves. Emission of these waves by one ferromagnetic spin and reabsorption by another also results in an extra exchange coupling among the ferromagnetic spins, but this is not calculated here in detail. A crucial test of the effectiveness of this mechanism as compared with others that have been proposed would be the observation of a reversal of the loop shift upon reversal of the magnetization of the ferromagnetic layer. However, the reversal time could be very long, and is estimated here. [S0163-1829(98)06126-8]

I. INTRODUCTION

When ferromagnetic films are deposited on an antiferromagnetic substrate under certain conditions, a shift in the hysteresis loop of the ferromagnetic film is observed so that it is no longer centered on zero applied field.[1] The effect, first observed over thirty years ago,[2] has elicited several alternative theoretical explanations.[3–5] The most recent of these[6,7] is based on a classical micromagnetic calculation, which, at least for certain relative alignments of the ferromagnetic and antiferromagnetic spins, can account for the effect. In this paper we note a further possible explanation based on quantum aspects of the distortion induced in the antiferromagnet spin system by its exchange coupling to the ferromagnetic spins across the interface. In turn, this distortion acts back on the ferromagnetic spins, resulting in an energy shift that, for spins greater than one-half, has the same form as a Zeeman energy, plus an extra anisotropy. The extra Zeeman field is responsible for the shift of the hysteresis loop, and its magnitude is in fair agreement with observation. The polarity of the extra Zeeman field is set by the orientation of the unperturbed ferromagnetic spin system; therefore its polarity should reverse upon reversal of the magnetization of the film. This would provide a crucial test of the theory. In Sec. IV the reversal process is discussed, and it is shown that its time dependence obeys a power rather than an exponential law, and the time scale for the reversal may be quite long.

II. THE MODEL

The spin arrangement on the antiferromagnetic side of the interface is assumed to be fully compensated;[8–11] thus in a plane normal to the interface the nominal spin configuration would appear as in Fig. 1. The Curie temperature of the ferromagnetic layer is assumed to be very much higher than the Néel temperature of the antiferromagnet. Then it is reasonable to suppose that the spin orientations in the ferromagnet are more robust than those of the antiferromagnet so that deviations of the true ground state from the nominal state shown in Fig. 1 are confined to the antiferromagnetic layer. In fact, the theory presented here is somewhat analogous to the Born-Oppenheimer theory[12] of electrons among ions: the spins of the ferromagnet play the role of the (initially fixed) ions, and the spins of the antiferromagnet play the role of the much more mobile electrons. The ground-state energy of the electrons depends parametrically on the assumed ionic positions, and at the next stage of the calculation, the quantum mechanics of the ions is governed by that energy surface. Analogously, here we calculate the distortion field of the antiferromagnetic layer assuming the ferromagnetic spins to be classical objects, and at the next stage consider the effect of the distortion on the dynamics of the latter.

Both spin systems are described by Heisenberg Hamiltonians. The exchange and anisotropy constants in these are assumed to be the same near and at the interface, as in the respective bulk media.

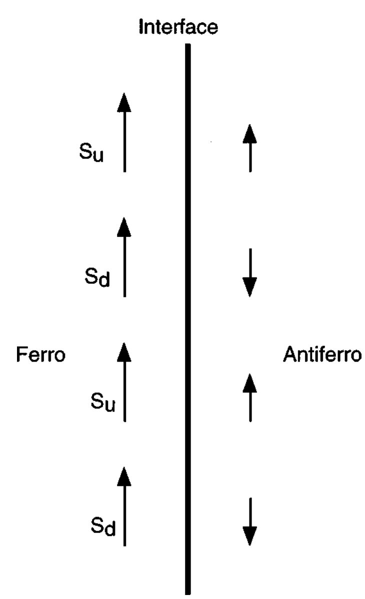

FIG. 1. Interface between the ferromagnetic and fully compensated antiferromagnetic layer modeled in the text.

III. THE HAMILTONIAN IN HOLSTEIN-PRIMAKOFF (HP) APPROXIMATION

The exact Hamiltonian is

$$H=H_{\mathrm{F}}+H_{\mathrm{AF}}+H_c+H_{an}$$
$$\equiv-\sum J_{\mathrm{F}}\mathbf{S}_i\cdot\mathbf{S}_j+\sum J_{\mathrm{AF}}\mathbf{s}_i\cdot\mathbf{s}_j+\sum J_c\mathbf{S}_i\mathbf{s}_j+\sum A(\mathbf{s}_i). \tag{1}$$

Upper and lower case S's, respectively, refer to ferromagnetic (F) and antiferromagnetic (AF) spins. The first two terms are obvious. The last term denotes the anisotropy energy in the AF layer and H_c is the coupling energy of the F and AF spins across the interface. The sign of J_c does not affect the results reported here. Note, however, that the signs of the terms in the third sum will alternate in the nominal (Néel) state, and will look different when deviations from the nominal state are taken into account. This is evident in the limit of large s, in which the HP transformation[13]

$$s_i^z=s-a_i^*a_i,\quad s_i^-=a_i^*\sqrt{2s},$$
$$s_i^+=a_i\sqrt{2s}\quad (i\ \text{an up-spin site})$$
$$s_i^z=-s+a_i^*a_i,\quad s_i^-=a_i\sqrt{2s},$$
$$s_i^+=a_i^*\sqrt{2s}\quad (i\ \text{a down-spin site})$$

is exact. (In the limit of large uniaxial anisotropy it is also exact for a different reason: the spins then cannot deviate from their Néel state orientations sufficiently to invalidate the HP transformation.) For small spins and small anisotropy the results derived here hold only in second-order perturbation theory. The a^*, a, respectively, create and destroy spin excitations of up spins, and do the opposite to down spins. They connect the nominal Néel state (or, for that matter, the exact ground state) to virtual excited states of the antiferromagnet. The F spins evidently act as sources of these excitations: they emit them, but also reabsorb them, resulting in a self-energy of each F spin (as well as a relatively small additional exchange interaction between F spins). Part of the self-energy has the form of Zeeman energy in a magnetic field whose numerical value is within an order of magnitude of the observed loop shift.

For discussing problems in the extended antiferromagnet, the a_i^*,a_i operators are conveniently expanded in a Fourier series, with coefficients a_k^*,a_k, defined by

$$a_k=\frac{1}{\sqrt{N}}\sum_i a_i e^{i\mathbf{k}\cdot\mathbf{r}_i},\quad a_i=\frac{1}{\sqrt{N}}\sum_k a_k e^{-i\mathbf{k}\cdot\mathbf{r}_i}. \tag{2}$$

For the semi-infinite medium, a small complication is that in the direction normal to the surface, the exponentials in the series must be replaced by combinations of sines and cosines. There may also be bound surface states. These complications are neglected here. Further, the exact ground state of H_{AF} is not the vacuum of the a_k operators, but of certain operators b_k, in terms of which

$$a_k=\cosh\theta_k b_k+\sinh\theta_k b_{-k}^*,$$
$$a_k^*=\cosh\theta_k b_k^*+\sinh\theta_k b_{-k}, \tag{3}$$

where

$$\tanh 2\theta_k=-2\gamma_k,\quad \gamma_k=\frac{1}{Z}\sum_{\mathrm{NN}} e^{i\mathbf{k}\cdot\boldsymbol{\delta}}.$$

The excitation energies are

$$(\hbar\omega_k)^2=CJ_{\mathrm{AF}}^2k^2+D^2$$

for low wave numbers. C is a constant incorporating the nearest-neighbor structure, and D^2 is a function of exchange and anisotropy energies (for the simple case of uniaxial anisotropy it is a linear sum of the square of the anisotropy and the product of exchange constant and anisotropy). The sum extends over nearest neighbors (NN), and $\boldsymbol{\delta}$ are the nearest-neighbor distances. The need to introduce the b operators arises from the fact that, besides terms such as $a_i^*a_i$ in H_{AF}, there are also terms such as aa and $a_i^*a_i^*$. These are removed by the transformation [Eq. (3)] to the new operators, in terms of which

$$H_{\mathrm{AF}}=\sum_k \hbar\omega_k b_k^* b_k+\text{const}.$$

In the present limit of large spin or large anisotropy, the resulting correction to the Néel ground state may be viewed as a zero-point motion of the spins, which slightly diminishes their z components. The effect of interest here is not much changed by that zero-point motion, Therefore, except in the crucial coupling terms of the AF and F spins at the interface, we shall for now ignore the difference between the a and b operators. (The difference is fully taken into account in Appendix B, which is concerned with the distortion of the ground-state AF spin distribution arising from coupling to the F layer.) In terms of the a's, the interaction looks different according to whether the AF neighbor of an F spin is up or down in the Néel state (see Fig. 1). So if we denote by $\mathbf{S}_u$ the uth F spin that has an AF up-spin neighbor, and by $\mathbf{S}_d$ the dth F spin that has an AF down-spin neighbor, then the interaction can be written

$$J_c\Bigg(\sum_{u,d}\{S_u^z(s-a_u^*a_u)+\sqrt{2s}[S_u^+a_u^*+S_u^-a_u]-S_d^z(s-a_d^*a_d)+\sqrt{2s}[S_d^+a_d+S_d^-a_d^*]\}\Bigg). \tag{4}$$

Of course, both $\mathbf{S}_u$ and $\mathbf{S}_d$ point in the *same* direction taken to be the up direction in Fig. 1, and they are equal in magnitude. In the ground state, the brackets $s-a_u^*a_u$ and $s-a_d^*a_d$ have the same values (diminished by zero-point motion). Therefore in the ground state, the first and third term cancel. However, in higher order, they change the excitation energies slightly (by a surface-to-volume ratio of the AF film), and also induce a small anisotropy in the ferromagnet. We neglect these terms hereafter. The second and fourth ''transverse'' terms in Eq. (4) have more important effects. In terms of the b operators, the transverse part of the Hamiltonian is

$$H_{\mathrm{tr}\,C}=\sqrt{\frac{2s}{N}}\sum_k\ (p_kb_k+p_k^*b_{-k}^*),\tag{5}$$

where

$$p_k=\sum_u\ (S_u^+\sinh\ \theta_ke^{-iku}+S_u^-\cosh\ \theta_ke^{iku})$$
$$+\sum_d\ (S_d^+\cosh\ \theta_ke^{ikd}+S_d^-\sinh\ \theta_ke^{-ikd}).\tag{6}$$

If the F spins are treated classically, this interaction may be viewed as shifting the origin of the harmonic oscillators described by the b's. This means that the vacuum of the system is no longer the vacuum of the b's. Instead, it is a vacuum of new harmonic oscillators with operators c_k^*,c_k, given by

$$b_k=c_k+\rho_k,$$

where the ρ_k are determined from the condition that the total Hamiltonian, expressed in terms of the c's, should no longer contain terms linear in the c's. Under this shift, the bulk part of the AF Hamiltonian becomes

$$\sum_k\ \omega_kb_k^*b_k\Rightarrow\sum_k\ \omega_kc_k^*c_k+\omega_k(c_k^*\rho_k+c_k\rho_k^*)+\omega_k\rho_k^*\rho_k\tag{7}$$

while the transverse interaction part becomes

$$H_{\mathrm{trans}}\Rightarrow J_0\sqrt{\frac{2s}{N}}\left\{\sum_k\ p_kc_k^*+p_k^*c_k+\sum_k\ p_k\rho_k^*+p_k^*\rho_k\right\}.\tag{8}$$

Equating to zero the coefficients of the c's in the sum of bulk and transverse Hamiltonians, we find

$$\rho_k=-J_0\sqrt{\frac{2s}{N}}\frac{p_k}{\omega_k}$$

and then the vacuum state of the c's has an energy equal to

$$E_{\mathrm{vac}}=-\frac{2s}{N}\ J_c^2\sum_k\ \frac{p_k^*p_k}{\omega_k}.\tag{9}$$

Although up to this point the F spins were treated as c numbers, we have taken care to preserve the order in which they appear in products of the p_k's. When the definition of the p's from Eq. (6) are substituted in Eq. (9), and only the self-interaction terms of the F spins are kept, the result is

$$-\frac{2sJ_c^2}{N}\sum_k\ \frac{1}{\omega_k}\left\{\left(\sum_u\ (S_u^+S_u^-\cosh^2\theta_k+S_u^-S_u^+\sinh^2\theta_k)\right)+\left(\sum_d\ (S_d^+S_d^-\sinh^2\theta_k+S_d^-S_d^+\cosh^2\theta_k)\right)\right\}.\tag{10}$$

At this point, we restore the operator aspects of the F spins, and use the relations

$$S_u^+S_u^-=2[S(S+1)-(S_u^z)^2+S_u^z],$$
$$S_u^-S_u^+=2[S(S+1)-(S_u^z)^2-S_u^z].\tag{11}$$

Combined with Eq. (10), this shows that, in the ground state of the ferromagnet (for which S^-S^+ is zero), an effective Zeemann field

$$g\mu_BH_{\mathrm{eff}}=\frac{2sJ_c^2}{N}\sum_k\ \frac{\cosh^2\theta_k}{\omega_k}\approx\frac{2sJ_c^2}{J_{\mathrm{AF}}N}\tag{12}$$

acts on the F spins opposite up AF spins, and a similar field, but with the cosh's replaced by sinh's act on F spins opposite to down AF spins. (There remains an ambiguity with regard to apportionment of this energy between the gyromagnetic ratio and the effective field.) If the F spins are greater than one-half, there is also an effective uniaxial anisotropy energy of a similar amount arising from the $(S_u^z)^2$ term, but we shall not examine it further here, since it does not give a unidirectional shift. The above calculation has been carried out at zero absolute temperature, but the temperature dependence must be very weak, except in a range of temperatures in which magnon-magnon scattering is significant, i.e., within a certain neighborhood of the Néel point.

So far, we have considered the self-energy of the superficial F spins only, which in itself would give a quite negligible loop shift. However, the effect spreads into the interior of the F layer. The reason is that the state with one F spin flipped in the emission process of the virtual AF spin wave is not an eigenstate of the F Hamiltonian. The correction to that state admixes flipped spins in the interior, for a certain distance. This distance is easily calculated in spin-wave approximation. It is equal to the lattice spacing times the square root of the ratio of exchange energy to anisotropy energy, and might be of the order of a thousand Angstroms. The reverse process also penetrates in this manner so that an S^+S^- occurs in that entire penetration distance.

IV. REVERSAL TIME

The only preferred direction in this problem is that of the ferromagnetic alignment. Therefore reversing the ferromagnetic lineup should reverse the exchange induced shift, once the system is allowed to come to equilibrium. Experimentally it does not reverse, at least not on the time scale of the experiments to date.

One possibility is that the field in which the system is cooled below T_N freezes in a preferred direction. In fact, mean-field theory shows that cooling in a field leads to a small net magnetization of the AF lattice. If this is unaffected by reversal of the F layer well below T_N, this could in principle break the symmetry. The difficulty with this explanation is that the small induced magnetization corresponds to a field of only $H_{\mathrm{applied}}\times(H_{\mathrm{applied}}/H_{\mathrm{exchange}})$, and for a T_N of 60 °K, an applied field of 1 kOe would make this number only about 20 Oe. Furthermore, in mean-field theory, this already small amount steadily decreases to zero as $T\to 0$. Thus the cooling field would have to do something more drastic that is not understood at present.

Conceivably the time needed for reversal of the exchange shift is longer than observation times used in experiments to date. Therefore we estimate the time needed by the system to adjust to a reversal of the ferromagnetic layer. We start with the ground state of the system in the presence of the shift, and use the HP approximation. We had found that the ground state is the vacuum of the operators

$$c_k = b_k - \rho_k . \tag{13}$$

We proceed by writing down this ground state as a superposition of eigenstates of the occupation numbers of the k states. Then we imagine the F layer suddenly removed at time $t=0$ (sudden approximation). Reversing the layer makes the problem more complicated without introducing a significantly different time scale (certainly not a shorter one). The resulting new wave function must be continuous with the old one at time 0. Since the states in the expansion of the old state are, in fact, eigenstates of $b_k^* b_k$, which are eigenstates of the new Hamiltonian with the F layer removed, the further time development is determined by appending the appropriate time factors $e^{it\omega_k}$ to the states in the series expansion of the old ground state. Using Eq. (13), and the recursion relations implied by it, one finds

$$|\mathrm{old}\rangle = \prod_k \sum_{n_k} \frac{\rho_k^{n_k}}{\sqrt{n_k!}} |n_k\rangle = \sum_{n_{k_1}, n_{k_2}, n_{k_3}, \ldots} \frac{\rho_{k_1}^{n_{k_1}} \rho_{k_2}^{n_{k_2}} \rho_{k_3}^{n_{k_3}} \cdots}{\sqrt{n_{k_1}! n_{k_2}! n_{k_3}! \cdots}} |n_{k_1}, n_{k_2}, n_{k_3} \ldots\rangle . \tag{14}$$

If now the F layer is switched off, at time $t=0$, this state will evolve to

$$\sum_{n_{k_1}, n_{k_2}, n_{k_3}, \ldots} \frac{\rho_{k_1}^{n_{k_1}} \rho_{k_2}^{n_{k_2}} \rho_{k_3}^{n_{k_3}} \cdots}{\sqrt{n_{k_1}! n_{k_2}! n_{k_3}! \ldots}} \times \exp[it(n_{k_1}\omega_{k_1} + n_{k_2}\omega_{k_2} + n_{k_3}\omega_{k_3} \ldots)] |n_{k_1}, n_{k_2}, n_{k_3} \ldots\rangle \tag{15}$$

at time t. Because $|n\rangle = ((b^*)^n/\sqrt{n!}\,)|\mathrm{vac}\rangle$, this result is more simply written

$$|t\rangle = \exp\left[\sum_k \rho_k e^{i\omega_k t} b_k^*\right] |\mathrm{vac}\rangle . \tag{16}$$

To see how, for example, the spin deviation $a_i^* a_i$ evolves in time, we make repeated use of the Baker-Hausdorff formula[14] in the form $e^{\lambda b} e^{\lambda^* b^*} = e^{(\lambda b + \lambda^* b^*) - \lambda\lambda^*/2}$. Thus we write Eq. (16) in the form

$$\begin{aligned} |t\rangle &= \exp\left[\sum_k \rho_k e^{i\omega_k t} b_k^*\right] \exp\left[\sum_k \rho_k^* e^{-i\omega_k t} b_k\right] |\mathrm{vac}\rangle \\ &= \exp\left[\sum_k (\rho_k e^{i\omega_k t} b_k^* + \rho_k^* e^{-i\omega_k t} b_k)\right] \\ &\quad \times \exp\left[\sum_k (1/2)\rho_k^* \rho_k\right] |\mathrm{vac}\rangle . \end{aligned} \tag{17}$$

Then, aside from a numerical factor, we get, writing $b_i = (1/\sqrt{N})\ \Sigma_k b_k e^{-i\mathbf{k}\cdot\mathbf{r}}$,

$$\begin{aligned} \langle t|b_i^* b_i|t\rangle &= \frac{1}{N} \sum_{k,k'} \langle t|b_{k'}^* b_k|t\rangle e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_i} \\ &= \frac{1}{N} \sum_{k,k'} \rho_{k'}^* \rho_k e^{i[(\omega_k - \omega_{k'})t + (\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_i]} \\ &= 2s \left| \frac{J_c}{N} \sum_k \frac{p_k e^{i[\omega_k t + \mathbf{k}\cdot\mathbf{r}_i]}}{\omega_k} \right|^2 \end{aligned} \tag{18}$$

and a very similar result for the spin deviations $a_i^* a_i$ themselves. For large times, and a quadratic spin-wave spectrum, this expression eventually decays like $1/t$, so there is no characteristic time associated with the decay. To see whether *macroscopic* times are involved, one needs to find the quasiscale t_0 in the expression (t_0/t), which must be estimated from Eq. (18). We have

$$\frac{1}{N} \sum_k \frac{p_k}{\omega_k} e^{i[\omega_k t + \mathbf{k}\cdot\mathbf{r}_i]} \Rightarrow \frac{V}{N} \int_V \frac{d^3k}{\omega_k} \sum_{u,d} (S_u^+ e^{-i\mathbf{k}\mathbf{u}} + S_d^- e^{-i\mathbf{k}\mathbf{d}}) e^{i[\omega_k t + \mathbf{k}\cdot\mathbf{r}_i]},$$

where V is the volume of the AF sample. For simplicity, we chose $r_i = 0$, and set $\omega_k = \omega_0 + J_{\mathrm{af}}\ell^2 k^2$. Integration over the angle between $\mathbf{k}$ and $\mathbf{u}$, etc., gives

$$\frac{4\pi V}{N} \int \frac{k^2 dk}{\omega_k} \sum_{u,d} (S_u^+ J_0(ku) + S_d^- J_0(kd)) e^{i\omega_k t}.$$

Setting $\sqrt{tJ_{\mathrm{af}}}\ell k = \kappa$ gives

$$\frac{4\pi e^{i\omega_0 t}}{(J_{\mathrm{af}})^{3/2} t^{1/2}} \int_0^\infty \frac{e^{i\kappa^2} \kappa^2 d\kappa}{\omega_0 t + \kappa^2} \sum_{u,d} \left[S_u^+ J_0\left(\frac{\kappa u}{\ell \sqrt{J_{\mathrm{at}} t}}\right) + S_d^- J_0\left(\frac{\kappa d}{\ell \sqrt{J_{\mathrm{af}} t}}\right) \right] \tag{19}$$

since $V/N = \ell^3$. For large t, the Bessel functions tend to unity. The remaining integral[15] has the form

$$\frac{1-i}{2} \sqrt{\frac{\pi}{2}} + \sqrt{\omega_0 t} \times \text{terms oscillating rapidly (at frequency } \omega_0). \tag{20}$$

Thus the total response consists of a term decaying like $t^{-1/2}$ plus a nondecaying but rapidly oscillating term. That term does not decay, because we have not included spin wave damping. On the other hand, the secular term is not much affected by the damping. The reason is that in the idealized model considered here, the damping must vary as $e^{-g\ell^2 k^2 t}$, where g is a constant, because the total spin almost commutes with the Hamiltonian, except for the anisotropy term, therefore the zero wave-number component cannot decay rapidly. Inversion symmetry then makes the decrement proportional to k^2, plus possibly higher powers of k. Now the secular term in Eq. (20) comes from the first term in the decomposition

$$\frac{\kappa^2}{\omega_0 t + \kappa^2} = 1 - \frac{\omega_0 t}{\omega_0 t + \kappa^2},$$

while the oscillating part comes from the second term. When the damping is included, that first term gives rise to the integral

$$\int_0^\infty e^{(i-\gamma/J_{\rm af})\kappa^2} d\kappa,$$

where γ is a damping constant. Since the ratio $\gamma/J_{\rm af}$ must be quite small, if the spin waves are to be viable modes, the effect of the damping on the secular term in Eq. (20) must be very small. Finally, we need to find the quasiscale of the secular decay, i.e., we must find the value of t_0 in the expression $(t_0/t)^{1/2}$. The secular part of the integral in Eq. (19), in the limit of large times, is

$$\frac{2\pi\sqrt{\pi}(1-i)}{J_{\rm af}\sqrt{2J_{\rm af}t}}\left(\sum_u S_u^+ + \sum_d S_d^-\right)$$

$$= \frac{2\pi\sqrt{\pi}(1-i)}{J_{\rm af}\sqrt{2J_{\rm af}t}}\frac{N_{\rm surf}}{2}(S_u^+ + S_d^-), \tag{21}$$

where $N_{\rm surf}$ is the total number of surface sites. Using this result in the last line of Eq. (18) gives the final result:

$$\frac{4\pi^3 N_{\rm surf} J_c^2}{J_{\rm af}^2(J_{\rm af}t)}\langle S_u^+ S_u^-\rangle.$$

(Note that the J's are expressed as frequencies.) To summarize, the secular decay goes like (t_0/t), where

$$t_0 = \frac{8\pi^3 N_{\rm surf}}{J_{\rm af}}\left(\frac{J_c}{J_{\rm af}}\right)^2 (S(S+1)-(S^z)^2+S^z). \tag{22}$$

This is a macroscopic time, because $J_{\rm af}\approx 80$ °K$\approx 10^{12}$ Hz. Also, $N_{\rm surf}$ might be in the range of 10^{15} for a 1 cm^2 sample. So if the coupling and AF J's are of the same order, the time scale becomes quite long.

V. CONCLUSIONS

The aim of this paper has been to demonstrate the importance of changes in the interface properties of widely disparate magnetic systems brought about by emission and reabsorption of virtual excitations. One consequence is the unidirectional shift in the hysteresis loop of a thin ferromagnetic layer in contact with an antiferromagnetic one. The same process also gives a change in anisotropy energy, and therefore in the coercive force, of the ferromagnetic layer (in the case of spins greater than one-half), but this change is not examined in detail in this paper. The calculated shift is in fair agreement with observed loop shifts reported in currently available experimental data. A detailed calculation for a different spin arrangement has recently been performed.[16] A crucial test of the validity of the theory is proposed: the loop shift should reverse upon reversal of the alignment of the ferromagnetic layer. The reversal process proceeds as $(\text{time})^{-1}$, and behaves like an extrinsic quantity in that it is sample size dependent.

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APPENDIX A: FORMAL DERIVATION OF THE RESULT OF SEC. III

Treating the F spins classically to begin with, and as quantum objects at the end, requires justification. Let P be the projection operator of the ground state of the Hamiltonian in the absence of coupling, and Q the projection operator of the manifold of its excited states. The total Hamiltonian is H_0+H_1, where H_1 denotes the off-diagonal part of the F-AF coupling energy, and H_0 the remainder. Then $PH_1P=0$, $QH_1P\neq 0$, $QH_0P=0$, etc. Schrödinger's equation

$$H(P+Q)|\ \rangle = E(P+Q)|\ \rangle$$

can be written as two equations

$$(PH_0P+PH_1Q)|\ \rangle = EP|\ \rangle$$

$$(QH_0Q+QH_1P)|\ \rangle = EQ|\ \rangle. \tag{A1}$$

From the second of these it follows that

$$Q|\ \rangle = \frac{1}{E-QH_0}QH_1P|\ \rangle.$$

Substituting this in the first equation of Eq. (A1) gives an equation for the ground state alone:

$$\left(H_0+H_1\frac{1}{E-QH_0}H_1\right)P|\ \rangle = EP|\ \rangle.$$

Evidently, because H_1 connects only to excited states, the Q in the denominator may be dropped. Unless the coupling is very strong, we may substitute the uncoupled ground-state energy, which we chose to be $E=0$, into the denominator on the left-hand side. Thus we may write

$$\left(H_0-H_1\frac{1}{H_0}H_1\right)P|\ \rangle = EP|\ \rangle. \tag{A2}$$

Finally we write the ground state as the product of ground states of the ferromagnetic and antiferromagnetic systems, respectively:

$$P|\ \rangle = |{\rm F}\rangle\times|{\rm AF}\rangle,$$

and then, to within the lowest Holstein-Primakoff approximation, Eq. (A2) becomes

$$\left(H_{0\rm F}-\sum_k \langle {\rm AF}|H_1|1_k\rangle\frac{1}{\omega_k+H_{0\rm F}}\langle 1_k|H_1|{\rm AF}\rangle\right)|{\rm F}\rangle = E|{\rm F}\rangle, \tag{A3}$$

where the sum extends over all states $|1_k\rangle$ with one AF spin wave excited, and H_{0F} pertains to the unperturbed Hamiltonian on the ferromagnetic site. The matrix elements in Eq. (A3) are still linear functions of the F spin operators. We can rewrite the sum in Eq. (A3) in the form

$$\sum_k \left[\langle AF|H_1|1_k\rangle\langle 1_k|H_1|AF\rangle \frac{1}{\omega_k+H_{0F}} + \langle AF|H_1|1_k\rangle \times\left(\frac{1}{\omega_k+H_{0F}}, \langle 1_k|H_1|AF\rangle \right)\right]. \tag{A4}$$

When this acts on $|F\rangle$, the denominator in the first term is simply $1/\omega_k$, since we assume the ground-state energy to be zero. To examine the second term, the commutator, assume, for simplicity, that the F spins have value one-half. Then

$$\frac{1}{\omega_k+H_{0F}} = a + a_1 \sum a(i,j)\mathbf{S}_i\cdot\mathbf{S}_j + \sum a(i,j,k)\mathbf{S}_i\cdot[\mathbf{S}_j\times\mathbf{S}_k]+\cdots,$$

in which the coefficients all have denominators with F excitation energies. The commutator is a similar sum (except that it has no constant term). Since the ferromagnetic array is assumed to be very stiff, the denominators are large, so that the second term in Eq. (A4) can be neglected compared with the first term, except possibly for contributions from very low-lying spin-wave excitations of the ferromagnet. These are probably excluded because of the large demagnetizing forces accompanying excitations of the F spins in a film geometry. The first term in Eq. (A4), written out in full, has precisely the form of Eq. (12) derived in the main text.

APPENDIX B: DISTORTION OF THE AF SPIN CONFIGURATION

The spin configuration in the new ground state of the antiferromagnetic side is found as follows. For brevity write $\cosh\,\theta_k \Rightarrow \cosh_k$, $\sinh\,\theta_k \Rightarrow \sinh_k$. We have, for an AF up spin in this state,

$$\begin{aligned}
\langle s_i^z\rangle &= s - \langle g|a_i^* a_i|g\rangle \\
&= s - \frac{1}{N}\sum_{k,k'} \langle g|(\cosh_k b_k^* + \sinh_k b_{-k})(\cosh_{k'} b_{k'} + \sinh_{k'} b_{-k'}^*)|g\rangle e^{i(\mathbf{k}'\cdot\mathbf{k})\cdot r_i} \\
&= s' - \frac{1}{N}\sum_{k,k'} (\rho_k^*\rho_{k'}\cosh_k\cosh_{k'} + \rho_{-k}\rho_{-k'}^*\cosh_k\cosh_{k'} + \rho_k^*\rho_{-k'}^*\cosh_k\sinh_{k'} + \rho_{-k}\rho_{k'}\cosh_{k'}\sinh_k) e^{i(\mathbf{k}'-\mathbf{k})\cdot\mathbf{r}_i} \\
&= s' - \frac{2sJ_0^2}{N^2}[S(S+1)+S^{z2}-S^z]\Bigg\{\sum_u \Bigg(\sum_k \frac{\sinh_k\cosh_k}{\omega_k} e^{-i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{u})}\sum_k \frac{\cosh_k^2}{\omega_k} e^{+i\mathbf{k}\cdot(\mathbf{r}_i+\mathbf{u})} \\
&\quad + \sum_k \frac{\sinh_k\cosh_k}{\omega_k} e^{-i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{u})}\sum_k \frac{\sinh_k\cosh_k}{\omega_k} e^{+i\mathbf{k}\cdot(\mathbf{r}_i+\mathbf{u})}\Bigg)\Bigg\}
\end{aligned} \tag{B1}$$

plus a similar sum over down spins, obtained from Eq. (B1) by interchanging $\sinh_k$ and $\cosh_k$, and replacing u by $-d$. Here, s' is the expectation value of the z component of a spin in the zero-point corrected ground state of the bulk antiferromagnet. We estimate the behavior of these sums only very roughly, by considering a typical sum, with all hyperbolic functions replaced by unity. A typical term of the sum over ''u'' sites then has the form

$$\langle s_i^z\rangle = s - 2s[S(S+1)+(S^z)^2-S^z]\left|\frac{J_0}{N}\sum_{d,k} e^{i\mathbf{k}\cdot(\mathbf{u}-\mathbf{r}_i)}/\omega_k\right|^2. \tag{B2}$$

For a quadratic spin-wave spectrum the sum over k in Eq. (8) is proportional to $e^{-\kappa|\mathbf{d}-\mathbf{r}_i|}/|\mathbf{u}-\mathbf{r}_i|$, where κ is proportional to the gap in the spin-wave spectrum. The remaining sum over the surface sites d, for a very large interface, of course depends only on the distance z into the AF layer. That sum, for a simple square lattice, is

$$\sum_{m,n} e^{-\kappa\sqrt{(m^2+n^2)\ell^2+z^2}}/\sqrt{(m^2+n^2)\ell^2+z^2},$$

where ℓ is the lattice spacing. In a continuum approximation to the surface this sum evaluates to $\approx e^{-\kappa z}$. A similar result applies to down-AF spins, except that there is an extra zero-point correction due to the reverse order of the a operators in the definition of s_z.

On the other hand, the transverse components of the AF spins average to zero in spite of the coupling to the ferromagnetic layer (which means that canting does not occur). The expectation value of s^+, for example, is proportional to the shift in an a or a^* operator. But the shift depends on $\langle S^+\rangle$ or $\langle S^-\rangle$ in the ferromagnetic states, and these expectation values are zero. Thus the F layer induces incoherent precessions in the AF spins within $1/\kappa$ of the surface. These precessions change the z components, but produce no average transverse components.

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